Math 120 B - Winter 2018
Midterm Exam Number Two
February 22nd, 2018

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	12	
2	12	
3	12	
4	12	
5	12	
Total	60	

- This exam consists of FIVE problems on THREE double-sided pages.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- Draw a box around your final answer to each problem.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the last page and indicate that you have done so. If you still need more room, ask your TA for an extra page to staple to your exam.
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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1. **[12 points]** My roommate is storing plums in an icebox. Their temperature (in Celsius) is an exponential function of time.

   4 hours from now, the plums will be at a temperature of $11°$ Celsius.

   16 hours from now, they will be at a temperature of $5°$ Celsius.

   (a) Write a function $p(t)$ for the temperature of the plums (in degrees Celsius) t hours from now.

$p(t) = A_0 b^t$

$A_0 b^4 = 11$

$A_0 b^{16} = 5$

$b^{12} = \frac{5}{11} \rightarrow b = \left(\frac{5}{11}\right)^{\frac{1}{12}}$

$A_0 \left(\frac{5}{11}\right)^{\frac{4}{12}} = 11 \rightarrow A_0 = \frac{11}{\left(\frac{5}{11}\right)^{\frac{4}{12}}}$

$A_0 \approx 14.31$

$$p(t) = 21.22 \left(\frac{5}{11}\right)^{\frac{t}{12}}$$

   (b) I will eat the plums when their temperature reaches $4.5°$ Celsius. How many hours from now is this?

$$14.31 \left(\frac{5}{11}\right)^{\frac{t}{12}} = 4.5$$

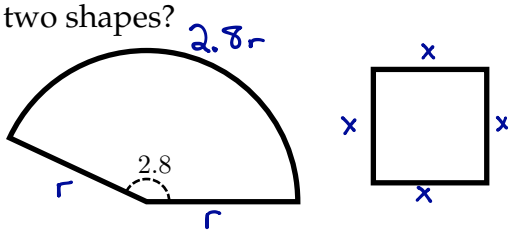
$$\left(\frac{5}{11}\right)^{\frac{t}{12}} = \frac{4.5}{14.31}$$

$$\frac{t}{12} \ln\left(\frac{5}{11}\right) = \ln\left(\frac{4.5}{14.31}\right)$$

$$t = \frac{12 \ln\left(\frac{4.5}{14.31}\right)}{\ln\left(\frac{5}{11}\right)} \approx 17.607 \text{ hours}$$

2. **[12 points]** You have 7.1 cm of wire that you will use to create two shapes: a sector with central angle 2.8 radians, and a square.

   What is the **minimum possible combined area** of the two shapes?



$A = \frac{1}{2}(2.8)r^2 + x^2$

$4.8r + 4x = 7.1$

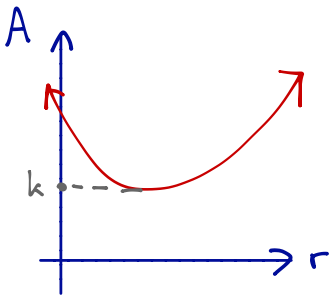
$4x = 7.1 - 4.8r$

$x = \frac{7.1}{4} - 1.2r$

$A = 1.4r^2 + \left(\frac{7.1}{4} - 1.2r\right)^2$

$= 1.4r^2 + \left(\frac{7.1^2}{4^2} - 4.26r + 1.44r^2\right)$

$= 2.84r^2 - 4.26r + 3.150625$



$k = 3.150625 - \frac{(-4.26)^2}{4(2.84)} \approx$ **1.553 sq. cm**

3. **[12 points]** As Zuri tends to his herb garden, the number of herbs it contains is a linear-to-linear rational function of time.

Right now, it contains 4 thousand herbs.

After 3 years, it will contain 4.8 thousand herbs.

In the long run, the number of herbs will approach (but not reach) 6 thousand.

(a) Write a linear-to-linear rational function $f(x)$ for the number of herbs (in thousands) in Zuri's garden after x years.

$f(x) = \frac{ax+b}{x+d}$

$a = 6$

$4 = \frac{b}{d} \rightarrow b = 4d$

$4.8 = \frac{3a+b}{3+d} \rightarrow 14.4 + 4.8d = 3a + b \rightarrow 14.4 + 4.8d = 18 + 4d$

$0.8d = 3.6$

$d = 4.5$

$b = 4d = 18$

$$f(x) = \frac{6x+18}{x+4.5}$$

(b) Find the inverse of the function you found in part (a).

$y = \frac{6x+18}{x+4.5}$

$xy + 4.5y = 6x + 18$

$xy - 6x = 18 - 4.5y$

$x(y-6) = 18 - 4.5y$

$x = \frac{18-4.5y}{y-6}$

$$f^{-1}(x) = \frac{18-4.5x}{x-6}$$

4. **[12 points]** Mirai has connected three wheels with an axle. A "triple axle", if you will.

   Wheel A has a radius of 5 inches and rotates at a linear speed of 12 inches per second.

   Wheel B has a radius of 3 inches.

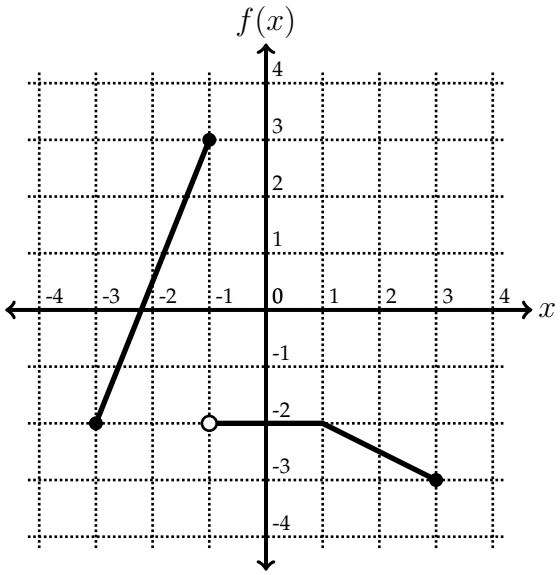
   Wheel C rotates 2 inches per second faster than wheel B.

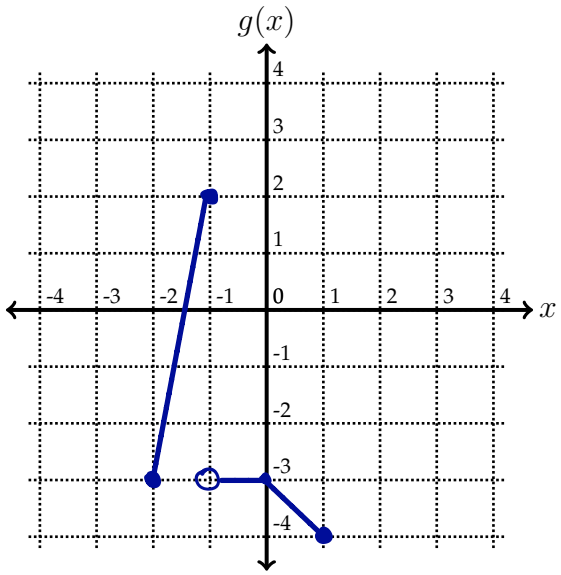
   What's the radius of wheel C?

Wheel	v (in./sec)	ω (rad./sec)	r (in.)
A	12	2.4	5
B	7.2	2.4	3
C	9.2	2.4	$\frac{9.2}{2.4}$

$$\frac{9.2}{2.4} = \frac{23}{6} \text{ in.}$$

5. **[12 points]** On the left is the graph of $f(x)$. On the right, please graph $g(x) = f(2x+1)-1$.





$y = f(2x+1) - 1$

$y+1 = f\left(\frac{x}{\frac{1}{2}} + 1\right)$

① Shift left 1
② Scale horiz. by $\frac{1}{2}$
③ Shift down 1.