Final topic review

The final exam is cumulative and covers the following topics.

1. Ch. 1: Introduction
   - What is a differential equation? Classification of differential equations.
   - How to check if a function solves a differential equation?
   - Direction field.
2. Ch. 2: First order DEs.
   - 2.2: Separable equations. Rewrite the DE in the form of $f(y)dy = g(x)dx$, and then integrate both sides.
   - 2.1: Method of integrating factors: Solve linear DE $\frac{dy}{dt} + p(t)y = q(t)$ by multiplying by the integrating factor $\mu(t) = e^{\int p(t)dt}$.
   - 2.3: Modeling/application problems. For example mixing problems, finance problems, and movements under gravity.
   - 2.4: Existence and uniqueness of solutions to IVP. Differentiate between linear and nonlinear DEs.
   - 2.5: Autonomous equations. Classification of equilibrium solutions by sketching the $t-y$ graph.
   - 2.7: Numerical approximation to solutions by Euler's method. $y_{k+1} = y_k + f(t_k, y_k) \cdot (t_{k+1} - t_k)$.
3. Ch. 3: Linear second order DEs.
   - 3.1, 3.3, 3.4: Homogeneous equation with constant coefficients $ay'' + by' + cy = 0$. Write down the general solution by looking at its characteristic equation.
   - 3.2: Principle of superposition. Fundamental sets of solutions. Wronskian determinant.
   - 3.4: Given a solution, using reduction of order to find another solution. Substitute in $y(t) = u(t)y_1(t)$ and find a DE for $u(t)$.
   - 3.5: Solutions to on-homogeneous equations are homogeneous solutions plus a particular solution. Using the method of undetermined coefficients to find a particular solution.
   - 3.7, 3.8: Mass spring system and electric circuit. Physical interpretation of the solutions.
4. Ch. 6: Using Laplace transform to solve linear DEs of any order.
   - 6.1: Computing the Laplace transform by definition. Inverse Laplace transform.
   - 6.2: Solving IVP with Laplace transform. Using derivative rules to solve for $\mathcal{L}y$, partial fractions, inverse Laplace transform to identify the solution.
   - 6.3: Laplace transform of step functions (first write piecewise continuous functions in terms of unit step function $u_c(t)$).
   - 6.4: Solving IVP with discontinuous forcing terms with Laplace transform.