Recent developments in noncommutative algebra and related areas

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ABSTRACT

Commutative-by-finite Hopf Algebras

Ken Brown

University of Glasgow, UK

I will define this class of Hopf algebras and discuss what is known about them, mostly old results, maybe a few new ones. There will be a number of questions (hopefully, open ones).

Factorization structures from the non-commutative Hilbert scheme of points

Emily Cliff

University of Illinois at Urbana-Champaign, USA

It follows from the work of Nakajima and Grojnowski that the cohomology of the Hilbert scheme of points on a surface X has the structure of a vertex algebra. Reinterpreting the axioms of a vertex algebra geometrically, we obtain for any smooth curve C a factorization algebra. The goal of this project is to construct the factorization algebra on C directly from the geometry of X and its Hilbert scheme, rather than via the language of vertex algebras. In this talk, I will discuss work-in-progress in this direction, using the Hilbert scheme of points on the three-fold X x C, as well as its non-commutative version. No prior knowledge of Hilbert schemes, vertex algebras, or factorization algebras will be assumed.

Catenarity and height formulas in quantum algebras

Ken Goodearl

University of California, Santa Barbara, USA

The prime ideals in large classes of quantum algebras – those of quantized coordinate ring type – share many properties in common with commutative affine algebras – coordinate rings of affine varieties. Particularly prevelant properties of such algebras A are *catenarity* (all saturated chains of prime ideals between any two prime ideals have the same length) and a *height formula* connecting heights to dimensions of quotients (such as $\text{height}(P) = \text{Kdim}(A) - \text{Kdim}(A/P)$). The talk will survey these and related ideas, known and new results, and conjectures.

Infinitely-categorified commutative algebra

Theo Johnson-Freyd

Perimeter Institute for Theoretical Physics, Canada

I will introduce an "infinite categorification" of commutative rings that I refer to as "towers" and that are a higher-categorical version of coconnective Ω-spectra. I will describe some basic constructions, including a "suspension" type construction that turns any commutative ring R or any symmetric monoidal linear category C into a tower $\Sigma^\bullet R$ or $\Sigma^{\bullet-1}C$. I will emphasize the role that finitely generated projective modules and that separable associative algebras play in these constructions. Through these, suspension towers are closely related to constructions in condensed matter and in topological field theory. I will end by suggesting an infinitely-categorified Galois theory, and in particular I will predict that the infinitely-categorified absolute Galois group of the real numbers is the stable orthogonal group. This talk is based in part on joint work with Davide Gaiotto.

Normal extensions of Artin-Schelter regular algebras and flat families of Calabi-Yau central extensions

Ryo Kanda

Osaka University, Japan

This is a joint work with Alex Chirvasitu and S. Paul Smith. We introduce a new method to construct 4-dimensional Artin-Schelter regular algebras as normal extensions of 3-dimensional ones. When this is applied to a 3-Calabi-Yau algebra, we obtain 4-Calabi-Yau algebras that form a flat family over a projective space. Our method is a rich source of new 4-dimensional regular algebras. Some of the 4-dimensional regular algebras discovered by Lu-Palmieri-Wu-Zhang also arise as outputs of our construction and our result gives a new proof of regularity for those algebras.

Reflection Hopf Algebras

Ellen Kirkman

Wake Forest University, USA

The Shephard-Todd-Chevalley Theorem states that when a finite group G acts linearly on a commutative polynomial ring $A = k[x_1, \ldots, x_n]$ over a field k of characteristic zero, the invariant subring A^G is a commutative polynomial ring if and only if G is generated by reflections. More generally, let H be a semisimple Hopf algebra that acts on an Artin-Schelter regular algebra A so that A is an H-module algebra, the grading on A is preserved, and the action of H on A is inner faithful. When A^H is Artin-Schelter regular, we call H a reflection Hopf algebra for A. We present examples of such pairs (A, H).

A smaller monoidal center for quantum group representations

Robert Laugwitz

Rutgers University, USA

The center of a monoidal category is an important construction in quantum algebra with applications to 3-dimensional TQFTs. This talk explains a center construction relative to a braided monoidal category which yields a smaller category. In the quantum group case, this recovers the monoidal category of highest weight representations with integral weights. The concept of a monoidal category relative to a braided monoidal category used here may be thought of as a categorification of the concept of a module algebra over a commutative ring.

Applications of the walks model of the colored Jones polynomial

Jesse Levitt

University of Southern California, USA

The colored Jones polynomial is a quantum knot invariant that plays a central role in low dimensional topology. We review a walks along a braid model of the colored Jones polynomial that was developed by Armond, Huynh and Le. The walk model gives rise to ordered words in a qWeyl algebra. We will discuss the faithfulness of this invariant with applications to the Jones unknot conjecture as well as several limiting behaviors with applications to the calculation of the Mahler measure of a knot. This is talk covers joint work with Mustafa Hajij and Nicolle E.S. Gonzales.

Twisted Matrix Factorizations and their Applications

Frank Moore

Wake Forest University, USA

Matrix factorizations were introduced by Eisenbud in 1980 in his study of homological properties of commutative hypersurface rings. In 2013, Cassidy, Conner, Kirkman, and the speaker generalized the definition of matrix factorization to the case of a normal regular element and used this definition to extend results that appeared in the original work of Eisenbud, as well as more recent work of Orlov. We will survey the known results on twisted matrix factorizations, mention some new results in this area, and also provide some questions that remain open.

Four-dimensional supersymmetric conformal field theories from six dimensions

Emily Nardoni

University of California, San Diego, USA

In physics, supersymmetric conformal field theories in six dimensions play an important role in our understanding of lower-dimensional field theories. We focus in this talk on the six-dimensional field theories with (2,0) supersymmetry, which are labeled by an ADE Lie algebra. We

discuss a construction of four-dimensional field theories by compactifying the six-dimensional theory on a punctured Riemann surface, with a particular embedding of the holonomy group of the Riemann surface in the structure group of the normal bundle to the spacetime manifold (a procedure known as a topological twist). The data of the four-dimensional theory depends on the ADE label of the parent six-dimensional theory, the details of the topological twist, and the data of the Riemann surface (genus and puncture data). We focus on a particularly interesting class of data known as anomalies, which are given as combinations of characteristic classes of the tangent and normal bundles to the spacetime manifold on which the theory is defined.

Stably noetherian algebras

Dan Rogalski

University of California, San Diego, USA

A noetherian algebra over a field k is called stably noetherian if it remains noetherian after the base field is extended to any other field. We survey some of the history of this problem and give some new results about which algebras satisfy this property. We also study variations of the problem where one considers only extensions of a certain type, for example purely transcendental extensions.

Formal deformation problems and the unicity of homotopy transfer

Chris Rogers

University of Nevada at Reno, USA

Suppose we are given a cochain complex A, a homotopy algebra B of some particular type (e.g., a homotopy associative, homotopy Lie, or homotopy Gerstenhaber algebra) and a quasi–isomorphism of complexes $\phi\colon A \to B$. Then a solution to the "homotopy transfer problem" is a pair consisting of a homotopy algebra structure on A, and a lift of ϕ to an equivalence of homotopy algebras $A \simeq B$.

I'll present a few results, some of which are based on joint work with Vasily Dolgushev, that explicitly relate the homotopy theory of dg Lie algebras to formal deformation problems. As an application, I will give an explicit construction of the moduli space (∞-groupoid) of solutions to the homotopy transfer problem. I will show that when

we are working over a field of characteristic 0 that: (1) the space of solutions is non-empty, and (2) it is contractible. The first statement implies the well-known fact that a homotopy equivalent transferred structure always exists, and the second implies that this structure is unique, up to homotopy, in the strongest possible sense (perhaps less well-known).

On the endomorphisms of some non-commutative algebras

Xin Tang

Fayetteville State University, USA

In a 1960 paper of Auslander and Goldman, it was proved that each endomorphism of an Azumaya algebra is an automorphism. In 1968, Dixmier completely determined the automorphism group for the first Weyl algebra (in characteristic zero) and asked whether each endomorphism of the first Weyl algebra is an automorphism. Dixmier's problem has been referred as the Dixmier Conjecture. The Dixmier conjecture has been proved to stably equivalent to the Jacobian conjecture. However, both conjectures remain largely open. Ever since, there has much interest in determining the automorphism group for a given algebra, establishing an analogue of the Dixmier Conjecture for an algebra, and finding criteria for an algebra endomorphism to be an automorphism. The first problem has been investigated by many mathematicians, and it has a vast literature beyond the scope of this talk. We do want to mention that there has been much progress made recently thanks to the rigidity theorem and the method of discriminants. In this talk, we will focus on the second and the third problems. After giving a brief survey on the research on these two problems, we will present some new results and examples in terms of establishing of an analogue of the Dixmier Conjecture and characterizing epimorphisms for a given algebra, which will hopefully generate future research interest.

Classification and invariants for fusion categories

Henry Tucker

University of California, San Diego, USA

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Weyl algebra (in characteristic zero) and asked whether each endomorphism of the first Weyl algebra is an automorphism. Dixmier's problem has been referred as the Dixmier Conjecture. The Dixmier conjecture has been proved to stably equivalent to the Jacobian conjecture. However, both conjectures remain largely open. Ever since, there has much interest in determining the automorphism group for a given algebra, establishing an analogue of the Dixmier Conjecture for an algebra, and finding criteria for an algebra endomorphism to be an automorphism. The first problem has been investigated by many mathematicians, and it has a vast literature beyond the scope of this talk. We do want to mention that there has been much progress made recently thanks to the rigidity theorem and the method of discriminants. In this talk, we will focus on the second and the third problems. After giving a brief survey on the research on these two problems, we will present some new results and examples in terms of establishing of an analogue of the Dixmier Conjecture and characterizing epimorphisms for a given algebra, which will hopefully generate future research interest.

Poisson geometry of PI elliptic algebras

Chelsea Walton

Temple University and University of Illinois at Urbana-Champaign, USA

I will present joint work with Xingting Wang and Milen Yakimov on the Poisson geometry and representation theory of various elliptic algebras that are module-finite over their center, as presented in the preprints
https://arxiv.org/abs/1704.04975 and
https://arxiv.org/abs/1802.06487v1.

Generalized Gorensteinness and Homological Determinants for Preprojective Algebras

Stephan Weispfenning

University of California, San Diego, USA

Studying invariant theory of commutative polynomial rings has motivated many developments in commutative algebra and algebraic geometry. The question under what conditions we can obtain a fixed ring with certain properties has been of particular interest. After generalizing the setting to certain noncommutative non-connected algebras, the main questions remain the same. This talk discusses a sufficient condition on the finite group acting to guarantee that the fixed ring has finite injective dimension and satisfies a generalized Gorenstein condition. Part of this result is the construction of a homological determinant of a non-connected algebra which turns out to be particularly nice for the examined preprojective algebras.