The Finite Dual of Commutative-by-finite Hopf Algebras

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joint work with Ken Brown & Astrid Jahn

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$k$ a field, $\bar{k} = k$, char $(k) = 0$

$(H, m, u, \Delta, \epsilon, S)$ Hopf algebra over $k$

$H^* = \text{Hom}(H, k)$

Finite dual:

$$H^\circ = \{ f \in H^* : f(I) = 0, \text{ for some } I \triangleleft H \text{ of finite codimension} \}$$

**Theorem (Kostant-Gabriel-Cartier)**

Let $H$ be an affine commutative Hopf algebra. Then, $H \cong \mathcal{O}(G)$ for some affine algebraic group $G$ and

$$H^\circ \cong U(g) \ast kG,$$

where

- $G \cong \text{Alg}(H, k)$ and $g = \text{Lie} G$.
- $U(g) = \{ f \in H^\circ : f((H^+)^n) = 0, \text{ for some } n > 0 \}$.
- $G$ acts on $U(g)$ by conjugation.
Affine commutative-by-finite Hopf algebras:

Affine $k$-Hopf algebras $H$ that are left (or right) module-finite over some normal commutative Hopf subalgebra $A$.

$A$ normal $\Rightarrow$ $A^+H$ Hopf ideal
$\Rightarrow$ $\overline{H} := H/A^+H$ Hopf algebra
Idea: decompose $H^\circ$ in terms of $\overline{H}^*$ and $A^\circ$.

**Theorem (B.-C.-J.)**

Let $H$ be an affine commutative-by-finite Hopf algebra. We write $H = A \oplus X$ as left (or right) $A$-modules.

1. If $X$ can be chosen to be a coideal of $H$, then as algebras

$$H^\circ \cong \overline{H}^* \ast A^\circ.$$ 

2. If $X$ can be chosen to be an ideal of $H$, then as algebras

$$H^\circ \cong \overline{H}^* \ast_\sigma A^\circ,$$

for some cocycle $\sigma$. 

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Finite dual of com.-by-fin. Hopf algebras

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\[ A^\circ \cong U(\mathfrak{g}) \ast kG, \]

- \( U(\mathfrak{g}) = \{ f \in A^\circ : f((A^+)^n) = 0, \text{ for some } n > 0 \} \).
- \( kG = \{ f \in A^\circ : f(\mathfrak{m}_{g_1} \cap \ldots \cap \mathfrak{m}_{g_r}) = 0, \text{ for some } \mathfrak{m}_{g_i} \triangleleft_{\text{max}} A \} \).

Under certain hypotheses,

\[ H^\circ \cong \overline{H}^* \ast_{\sigma} (U(\mathfrak{g}) \ast kG) \]

\[
\begin{array}{c}
W = \overline{H}^* \ast_{\tau} U(\mathfrak{g}) \\
\overline{H}^* \ast_{\sigma} kG = \widehat{kG}
\end{array}
\]
References


Thank you.