Projective Geometry Associated to some Quadratic Quantum $\mathbb{P}^3$s

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Recent Developments in Noncommutative Algebra and Related Areas

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What do I do?

- Analyze “noncommutative analogues” of poly. ring on 4 gens.
- Examine lines in $\mathbb{P}^3$ that corresp. to line mods. over these algebras.
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- Analyze “noncommutative analogues” of poly. ring on 4 gens.
- Examine lines in \( \mathbb{P}^3 \) that correspond to line mods. over these algebras.

Why?

- Point modules of \( A \) are parametrized by a scheme (the point scheme) ([ATV1] \( \sim \) 1990).
- Such schemes have led to the classification of these “noncommutative analogues” when \( n \leq 3 \).
- With a few more hyps., the line modules of \( A \) are parametrized by a scheme (the line scheme, \( \mathbb{L}_A \)) ([SV1] \( \sim \) 2000).

Goal: Shed light on the case when \( n = 4 \).
For some Graded skew Clifford Algebras

Field $\mathbb{k} = \overline{k}$ with $\text{char}(\mathbb{k}) = 0$.

**Line Scheme for $\mathcal{A}_{EE}$ (T.)**

For almost all algs. in family studied, $\mathbb{L}_{EE} = \text{union of 7 subschemes in } \mathbb{P}^5$:

- 3 nonplanar elliptic curves (each of degree 4)
- 4 nonsingular conics (each of degree 2).

$\mathbb{L}_{EE}$ has 24 distinct intersection points.

**Line Scheme for $\mathcal{A}(\alpha)$ (T. & Vancliff)**

For almost all algs. of family studied, $\mathbb{L}_{\alpha} = \text{union of 6 subschemes in } \mathbb{P}^5$:

- 1 nonplanar elliptic curve in a $\mathbb{P}^3$ (degree 4),
- 1 nonplanar rational curve in a $\mathbb{P}^3$ (degree 4),
- 2 planar elliptic curves (each of degree 3),
- 2 subschemes, each consisting of union of a nonsingular conic and a line (each of degree 3).
Theorem – T. & Vancliff

Let \( \delta, \epsilon \in k^\times \), and let \( N = \text{span of the norm. seq.} \)
\[
\{x_2^2, x_3^2, x_3x_4 + \delta x_4x_3, x_1x_2 + \epsilon x_2x_1\}
\]
of \( A(\alpha) \).

If \( p = \text{int. pt. of } \mathbb{I}_{A(\alpha)} \) and \( J_p = \text{right ideal of } A(\alpha) \) assoc. to \( p \), then
\[
\dim_k (N \cap J_p) = 2.
\]
**Theorem – T. & Vancliff**

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$$\{x_2^2, x_3^2, x_3x_4 + \delta x_4x_3, x_1x_2 + \epsilon x_2x_1\}$$ of $A(\alpha)$.

If $p = \text{int. pt. of } \mathbb{L}_A(\alpha)$ and $J_p = \text{right ideal of } A(\alpha)$ assoc. to $p$, then

$$\dim_k (N \cap J_p) = 2.$$ 

**Much different direction:** Coalgebras have comodules – can we construct *point comodules*? Or *d-linear comodules*?
Theorem – T. & Vancliff

Let \( \delta, \epsilon \in \mathbb{k}^\times \), and let \( N = \text{span of the norm. seq.} \)
\[
\{ x_2^2, x_3^2, x_3x_4 + \delta x_4x_3, x_1x_2 + \epsilon x_2x_1 \} \quad \text{of} \quad \mathcal{A}(\alpha).
\]

If \( p = \text{int. pt. of } \mathbb{L}_{\mathcal{A}(\alpha)} \) and \( J_p = \text{right ideal of } \mathcal{A}(\alpha) \) assoc. to \( p \), then
\[
\dim_{\mathbb{k}} (N \cap J_p) = 2.
\]

Much different direction: Coalgebras have comodules – can we construct point comodules? Or d-linear comodules?

Thank you all very much for being here!
References


