

# On the Adjoint Representation of Hopf Algebras

On the adjoint  
representation  
of  
Hopf algebras

Adam Jacoby

Motivation

Hopf  
annihilator

Conjugacy  
classes

## On the adjoint representation of Hopf algebras

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# Outline

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1 Background and motivation

2 The Hopf annihilator of the adjoint representation

3 Conjugacy classes

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# Notation

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Throughout the talk we will use the following notation.

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- $\mathbb{K}$  will be a field with  $\text{char } \mathbb{K} = p \geq 0$
- $(\cdot)^* := \text{Hom}_{\mathbb{K}}(\cdot, \mathbb{K})$  will denote the  $\mathbb{K}$ -linear dual
- $G$  will denote a finite group
- $H$  will denote an arbitrary Hopf  $\mathbb{K}$ -algebra
- $(\cdot)^+ := \ker \epsilon$  will denote the augmentation ideal

# The adjoint representation of a group

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A group  $G$  acts on its self by conjugation.

$${}^g h = ghg^{-1} \quad (g, h \in G)$$

Extending  $\mathbb{K}$ -linearly gives an action of  $\mathbb{K}G$  on itself.

## Definition.

The group algebra equipped with this action will be called the *adjoint representation*, denoted  ${}^{\text{ad}}\mathbb{K}G$ .

# The group picture

On the adjoint  
representation  
of  
Hopf algebras

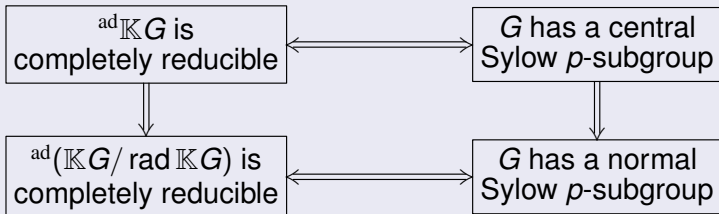
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## Theorem.



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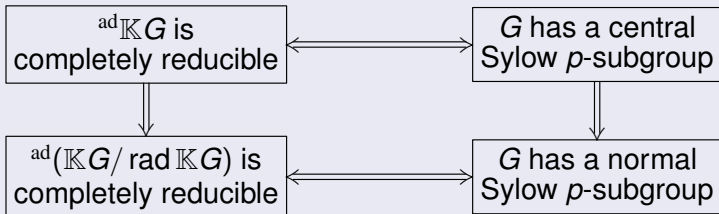
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## Theorem.



## Definition.

A module  $V$  has the *Chevalley property* if  $T(V) := \bigoplus_{n \in \mathbb{N}} V^{\otimes n}$  is completely reducible.



# The group picture

On the adjoint  
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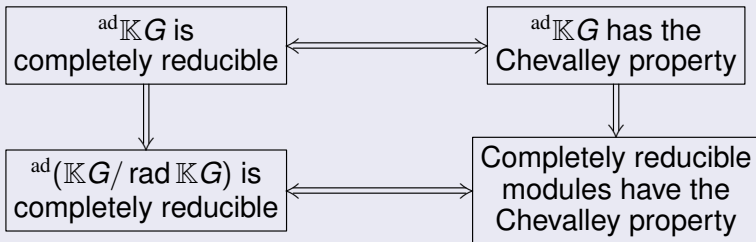
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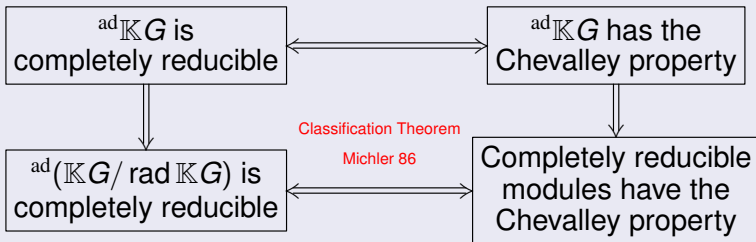
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# Components of the proof of the top implication

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## Sketch of proof

- 1 The largest Hopf ideal of  $\mathbb{K}G$  that annihilates  ${}^{\text{ad}}\mathbb{K}G$  is:

$$\mathbb{K}G(\mathbb{K}\mathcal{L}(G))^+$$

# Components of the proof of the top implication

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- 3 (2) implies  $p$  does not divide  $|G/\mathcal{L}(G)|$

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- 3 (2) implies  $p$  does not divide  $|G/\mathcal{L}(G)|$
- 4 (3) implies  ${}^{\text{ad}}\mathbb{K}G$  has the Chevalley property

# Components of the proof of the top implication

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## Sketch of proof

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# Definitions and notation

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A Hopf algebra  $H$  acts on its self via the adjoint action.

$${}^h k = h_{(1)} k S(h_{(2)}) \quad (h, k \in H)$$

## Definition

The Hopf algebra equipped with this action will be called the *adjoint representation*, denoted  ${}^{\text{ad}} H$ .

---



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- For  $I \leq H$  an ideal,  $\mathcal{H} I$  will denote the largest Hopf ideal contained in  $I$ .
- For  $A \subseteq H$  a subalgebra,  $\mathcal{H} A$  will denote the largest Hopf subalgebra contained in  $A$ .

# Definitions and notation

A Hopf algebra  $H$  acts on its self via the adjoint action.

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## Definition

The Hopf algebra equipped with this action will be called the *adjoint representation*, denoted  ${}^{\text{ad}} H$ .

- For  $I \leq H$  an ideal,  $\mathcal{H} I$  will denote the largest Hopf ideal contained in  $I$ .
- For  $A \subseteq H$  a subalgebra,  $\mathcal{H} A$  will denote the largest Hopf subalgebra contained in  $A$ .
- Let  $\zeta(H)$  denoted the largest Hopf subalgebra contained in the center of  $H$ .

# The Hopf annihilator of the adjoint representation

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## Theorem 1. (J.)

Let  $H$  be a Hopf algebra that satisfies one of the following conditions:

- 1  $H$  is finite-dimensional or
- 2 the coradical of  $H$  is cocommutative (e.g.,  $H$  is cocommutative or pointed).

Then the Hopf annihilator of the adjoint representation is given by  $\mathcal{H}(\text{ann}^{\text{ad}} H) = H\zeta(H)^+$ .

# The proof: Coinvariants I

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- Let  $\overline{H} = H/\mathcal{H}(\text{ann}^{\text{ad}} H)$

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- Let  $\bar{H} = H / \mathcal{H}(\text{ann}^{\text{ad}} H)$
- $H$  becomes a left  $\bar{H}$ -comodule via  $(- \otimes \text{Id}) \circ \Delta : H \rightarrow \bar{H} \otimes H$  i.e.  $h \mapsto \bar{h}_{(1)} \otimes h_{(2)}$

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- Let  ${}^{\text{co}\bar{H}}H := \{h \in H \mid \bar{h}_{(1)} \otimes h_{(2)} = \bar{1} \otimes h\}$

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- Let  ${}^{\text{co}\bar{H}}H := \{h \in H \mid \bar{h}_{(1)} \otimes h_{(2)} = \bar{1} \otimes h\}$
- $\zeta(H)^+ \subseteq \mathcal{H}(\text{ann}^{\text{ad}} H)$  since
$${}^z h = z_{(1)} h S(z_{(2)}) = z_{(1)} S(z_{(2)}) h = \epsilon(z) h = 0$$



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- Let  ${}^{\text{co}\bar{H}}H := \{h \in H \mid \bar{h}_{(1)} \otimes h_{(2)} = \bar{1} \otimes h\}$
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$${}^z h = z_{(1)} h S(z_{(2)}) = z_{(1)} S(z_{(2)}) h = \epsilon(z) h = 0$$
- Now  $\zeta(H) \subseteq {}^{\text{co}\bar{H}}H$  since
$$\begin{aligned}\bar{z}_{(1)} \otimes z_{(2)} &= \overline{(z_{(1)} - \epsilon(z_{(1)})\bar{1} + \epsilon(z_{(1)})\bar{1})} \otimes z_{(2)} \\ &= \epsilon(z_{(1)})\bar{1} \otimes z_{(2)} = \bar{1} \otimes z\end{aligned}$$

# The proof: Coinvariants II

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- ${}^{\text{co}H}H \subseteq \mathcal{L}(H)$  since

$$\begin{aligned}ch &= c_{(1)}h\epsilon(c_{(2)}) = c_{(1)}hS(c_{(2)})c_{(3)} \\ &= {}^{c_{(1)}}hc_{(2)} = \bar{c}_{(1)}hc_{(2)} = \bar{1}hc = hc\end{aligned}$$

# The proof: Coinvariants II

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- $co\bar{H}H \subseteq \mathcal{L}(H)$  since

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- $co\bar{H}H$  is a right subcomodule of  $H$ , thus:

$$\Delta(co\bar{H}H) \subseteq co\bar{H}H \otimes H \subseteq \mathcal{L}(H) \otimes H$$

# The proof: Coinvariants II

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- ${}^{coH}H \subseteq \mathcal{Z}(H)$  since

$$\begin{aligned}ch &= c_{(1)}h\epsilon(c_{(2)}) = c_{(1)}hS(c_{(2)})c_{(3)} \\ &= {}^{c_{(1)}}hc_{(2)} = \bar{c}_{(1)}hc_{(2)} = \bar{1}hc = hc\end{aligned}$$

- ${}^{coH}H$  is a right subcomodule of  $H$ , thus:

$$\Delta({}^{coH}H) \subseteq {}^{coH}H \otimes H \subseteq \mathcal{Z}(H) \otimes H$$

**Theorem.** (Chirvasitu, Kasprzak. preprint)

$$\zeta(H) = \{h \in H \mid \Delta(h) \in \mathcal{Z}(H) \otimes H\}$$

# The proof: Coinvariants II

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- $co\bar{H}H \subseteq \mathcal{Z}(H)$  since

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- $co\bar{H}H$  is a right subcomodule of  $H$ , thus:

$$\Delta(co\bar{H}H) \subseteq co\bar{H}H \otimes H \subseteq \mathcal{Z}(H) \otimes H$$

**Theorem.** (Chirvasitu, Kasprzak. preprint)

$$\zeta(H) = \{h \in H \mid \Delta(h) \in \mathcal{Z}(H) \otimes H\}$$

- Giving  $co\bar{H}H \subseteq \zeta(H)$  and so  $co\bar{H}H = \zeta(H)$

# The proof: faithful (co)flatness

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Recall the assumption of Theorem 1 that:

- 1  $H$  is finite-dimensional or
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# The proof: faithful (co)flatness

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Recall the assumption of Theorem 1 that:

- 1  $H$  is finite-dimensional or
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Either imply  $H$  is a faithfully coflat  $\overline{H}$ -comodule. Thus:

# The proof: faithful (co)flatness

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- $H$  is a faithfully flat  $\zeta(H)$ -module
- $H$  is a faithfully coflat  $H/H\zeta(H)^+$ -comodule



# The proof: an equivalence

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**Theorem.** (Takeuchi 79)

We have the following inverse maps:

$$\left\{ A \mid \begin{array}{l} \text{a left } H\text{-comodule algebra} \\ H \text{ faithfully flat over } A \end{array} \right\} \begin{array}{c} \xrightarrow{\text{co}H/IH} \\ \xleftarrow{HA^+} \end{array} \left\{ I \mid \begin{array}{l} I \text{ left } H\text{-module coideal} \\ H \text{ faithfully coflat over } H/I \end{array} \right\}$$

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The result follows from the diagram below:

$$\begin{array}{ccc} & & H\zeta(H)^+ \\ & \nearrow & \parallel \\ \zeta(H) & \longleftarrow & \mathcal{H}(\text{ann}^{\text{ad}} H) \end{array}$$

□

# Consequences

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For the remainder assume  $H$  is finite-dimensional.

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For the remainder assume  $H$  is finite-dimensional.

**Theorem.** (Rieffel 67)

For  $V$  an  $H$ -module:

$$\text{ann } T(V) = \mathcal{H}(\text{ann } V)$$

Thus  $V$  has the Chevalley property iff  $H/(\mathcal{H} \text{ann } V)$  is semisimple.

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**Corollary 1.** (J.)

${}^{\text{ad}}H$  has the Chevalley property iff  $H/H\zeta(H)^+$  is semisimple.

# Consequences

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**Corollary 1.** (J.)

${}^{\text{ad}}H$  has the Chevalley property iff  $H/H\zeta(H)^+$  is semisimple.

**Corollary 2.** (J.)

${}^{\text{ad}}H$  has the Chevalley property implies  $H$  is unimodular.

# Review: Drinfeld double

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Now  ${}^{\text{ad}}H$  can be viewed as a right  $H$ -comodule with structure map  $\Delta$ . With this  ${}^{\text{ad}}H$  becomes a Yetter-Drinfeld module, thus it is natural to consider the Drinfeld double.

# Review: Drinfeld double

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Now  ${}^{\text{ad}}H$  can be viewed as a right  $H$ -comodule with structure map  $\Delta$ . With this  ${}^{\text{ad}}H$  becomes a Yetter-Drinfeld module, thus it is natural to consider the Drinfeld double.

## Definition.

The *Drinfeld double* of  $H$  is the Hopf algebra  $D(H)$ . The coalgebra structure of  $D(H)$  is given by:

$$D(H) \stackrel{\text{coalg}}{\cong} H^{*\text{cop}} \otimes H$$

The element  $f \otimes h$  is denoted  $f \bowtie h$ . The multiplication is given by:

$$(f \bowtie h)(g \bowtie k) = f(h_{(1)} \rightharpoonup g \leftarrow S^{-1}(h_{(3)})) \bowtie h_{(2)}k$$



# Conjugacy class definition

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The Drinfeld double acts on  $H$  via the action below:

$$(f \bowtie h).k = ({}^h k) \leftarrow S^{-1}(f) \quad (f \in H^*, k \in H)$$

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$$(f \bowtie h).k = ({}^h k) \leftarrow S^{-1}(f) \quad (f \in H^* h, k \in H)$$

**Definition.** (Cohen, Westreich 2010)

If  $H$  is a completely reducible  $D(H)$ -module then we say a *conjugacy class* is a simple  $D(H)$ -submodule of  $H$ .

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**Definition.** (Cohen, Westreich 2010)

If  $H$  is a completely reducible  $D(H)$ -module then we say a *conjugacy class* is a simple  $D(H)$ -submodule of  $H$ .

**Example: group algebras**

The action of  $D(\mathbb{K}G)$  on  $\mathbb{K}G$  is completely reducible. The conjugacy classes, as defined above, are the modules arising from  $D(\mathbb{K}G)$  acting as above on the  $\mathbb{K}$ -span of the classical conjugacy classes.

# Results on conjugacy classes

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## Proposition 1. (J.)

For  $H$  a finite-dimensional Hopf algebra:

- 1 If  $H$  is a completely reducible  $D(H)$ -module then  $H$  is cosemisimple.
- 2 If  $H$  is cosemisimple and  ${}^{\text{ad}}H$  is a completely reducible then  $H$  is a completely reducible  $D(H)$ -module.

# Results on conjugacy classes

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## Proposition 1. (J.)

For  $H$  a finite-dimensional Hopf algebra:

- 1 If  $H$  is a completely reducible  $D(H)$ -module then  $H$  is cosemisimple.
- 2 If  $H$  is cosemisimple and  ${}^{\text{ad}}H$  is a completely reducible then  $H$  is a completely reducible  $D(H)$ -module.

## Theorem 3. (J.)

Let  $H$  be a cosemisimple, involutory Hopf algebra with  $\mathbb{K} = \overline{\mathbb{K}}$  then  ${}^{\text{ad}}H$  completely reducible implies  $\text{char } \mathbb{K}$  does not divide the dimension of any of the conjugacy classes.

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