Final Exam–Math 126 E/F, Spring 2018

The final Exam will be given on **Saturday, June 2, 2018** from 5:00-7:50pm. It will cover Taylor Notes, Ch. 10, 12, 13, 14 and 15. The exam room is **KNE 210** for Section E and **KNE 220** for Section F.

Suggestions: Use old final exams as practice tests.

https://sites.math.washington.edu/~m126/finals/final.php

Some basic rules

1. You are allowed to use a TI-30X IIS calculator. But **NO** other calculators are allowed.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work. You will not receive any (partial) credit unless all work is clearly shown.
4. Give your answers in exact form. For example, 3π, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are eight questions in the exam. Each question contains several parts.
6. Different topics could be combined in one question in the final exam.
7. Place a box around your final answer to each question.

Review topics

0: See Review 1 and Review 2.

1: Taylor series and operations with Taylor Series.

2: Taylor polynomials, approximation, error bounds, finding M, n, I etc.

3: Mass, moments and the center of Mass. Double integrals in polar coordinates.

4: Double integrals, volume/area.

Practice problems for Taylor polynomials and Taylor Series

1. Consider the function

$$f(x) = \frac{x}{1-x^2} - \int_0^x e^{t^2}\,dt.$$

(a) Find the Taylor series for $f(x)$ based at $b = 0$. (b) Give the open interval of convergence for the Taylor series in part (a). (c) What are $f^{(2018)}(0)$ and $f^{(2019)}(0)$?

2. Find the Taylor series for $\frac{1}{2-x}$ based at $b = 3$.

3. Consider the function

$$f(x) = 3\cos(2x) - \frac{\sin x}{x} + \frac{3}{1-x^2}.$$

(a) Find the Taylor series for $f(x)$ based at $b = 0$. (b) Find the first four nonzero terms in part (a). (c) Give the open interval of convergence for the Taylor series in part (a).

4. Consider the function

$$f(x) = x^2 e^x$$

(a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 1$. (b) Use the Taylor inequality to bound the error on the interval $I = [0.9, 1.1]$. (c) What is the smallest value of $|f(x) - T_2(x)|$ on the interval I.

5. Consider the function

$$f(x) = \sin(x^2 - 1)$$

(a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 1$. (b) Use the second Taylor polynomial $T_2(x)$ to approximate $f(1.01)$. (c) Use the Taylor inequality to find an interval J containing b so that the error bound is at most 0.001.

Practice problems for double integrals

1. Find the center of mass of the lamina that occupies the region

$$D = \{(x, y) \quad : \quad x^2 + y^2 \leq 4\}$$

with density function $\rho(x, y) = 1 + (x^2 + 1)\sqrt{x^2 + y^2}$.

2. Find the volume of the solid enclosed by the hyperboloid $-x^2 - y^2 + z^2 = 16$ and the plane $z = 5$.

3. Consider the cardioid given by the polar function $r = 2 - 2\cos(\theta)$. Set up a double integral in polar coordinates that represents the area inside this cardioid and outside the circle centered at the origin with radius 2.

4. Compute

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\,dx\,dy.$$