## Midterm two–Math 126 E/F, Spring 2018

Midterm two will be given on Thursday May 3 in quiz section (50 minutes only). It will cover Chapters 13 and 14.

### Some basic rules

1. You are allowed to use a TI-30X IIS calculator. But **NO** other calculators are allowed.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work. You will not receive any (partial) credit unless all work is clearly shown. Always simplify your final answers.
4. Unless otherwise stated, always give your answers in exact form. For example, $3\pi$, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are 4 questions in the exam. Each contains several parts.
6. No make-up exams.

### Practice problems

**Problem 1:** (Sections 13.1-13.3) Parametrization of 3-D curves, vector functions, derivative and integral of vector functions, arclength, curvature, $TNB$ system, normal (and osculating) plane.

**Example:** Let $\overrightarrow{\mathbf{r}}(t) = \langle 4t + 100, 3\sin(t-3), 3\cos(t-3)\rangle$ be the position function.

(a) Find the derivative of $\overrightarrow{\mathbf{r}}(t)$.

(b) Find the arc length of the curve from $t = 3$ to $t = 3 + \pi$.

(c) Find the curvature at any time $t$.

(d) Find $TNB$ at any time $t$.

(e) Find the tangential and normal components of the acceleration. (This part will be in Problem 2.)

(f) Find the normal plane at $t = 3$.

**Problem 2:** (Section 13.4) Newton's Second law of Motion, velocity and acceleration, speed, tangential and normal components of the acceleration.

**Example:** A ball is thrown at an angle of 45° to the the ground. If the ball lands 90m away, what was the initial speed of the ball?

**Problem 3:** (Sections 14.1-14.4) Functions of two (or several) variables, surfaces and level curves (contour map), partial derivatives, higher order partial derivatives, tangent plane/linear approximation.

**Examples:**

(1) Let $f(x, y) = x\ln(y + x) + e^{xy} - 3x + 2y^2$. Find $f_{xx}$, $f_{xy}$, $f_{yx}$ and $f_{yy}$.

(2) Let $f(x, y) = e^{\sin(x)-2y} + \tan(x^2 - y)$. Find the tangent plane to the surface at $(0, 0)$. Use it to approximate $f(0.1, -0.2)$.

(3) Find the equation of the tangent plane to the surface defined by the equation $xy + z^{2018} = 3$ at the point $(1, 2, 1)$.

**Problem 4:** (Section 14.7) Critical points, maximum/minimum values, first derivative test, second derivative test, saddle points, applications.

**Examples:**

(1) Find the points on the surface $2x^3y+z^2 = 4$ that are closest to the origin.

(2) Consider the function $f(x, y) = x^2 + y^2 - xy$ over $D$, where $D$ is the region enclosed by the circle of radius 4 centered at the origin.

(a) Find and classify all critical points.

(b) Find the absolute maximum value of $f(x, y)$ over $D$.

(3) Find nonnegative numbers $x$, $y$ and $z$ that minimize the quantity

$$M = x^2 + y^2 + z^2$$

subject to the condition

$$xy^2z^4 = 1.$$

(4) Find and classify all critical points of $z = (x^2 + y^2)e^{-x}$.

**Note that problems in the exam may not be in this order and that practice problems may not occur in the exam.**

**Please review all homework problems.**

**Usually Exam 2 is harder than Exam 1.**

Old Exam1 and Exam2 can be found at
https://sites.math.washington.edu/∼m126/midterms/midterm1.php
and
https://sites.math.washington.edu/∼m126/midterms/midterm2.php