**Midterm one–Math 126 E/F, Spring 2018**

Midterm one will be given on Thursday April 12 in quiz section (50 minutes only). It will cover Sections 10.3 and 12.1-12.6.

**Some basic rules**

1. You are allowed to use a TI-30X IIS calculator. But **NO** other calculators are allowed.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work. You will not receive any (partial) credit unless all work is clearly shown. Always simplify your final answers.
4. Unless otherwise stated, always give your answers in exact form. For example, $3\pi$, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are 4 questions in the exam. Each contains several parts.
6. No make-up exams.

**Practice problems**

**Problem 1:** (Sections 12.1-12.4) You need know vectors, dot and cross products, etc.

**Example 1:** Let $\overrightarrow{\mathbf{u}} = \langle 1, 2, 3\rangle$ and $\overrightarrow{\mathbf{v}} = \langle -2, -1, 7\rangle$.

(a) Find $\overrightarrow{\mathbf{u}} \bullet \overrightarrow{\mathbf{v}}$ and $\overrightarrow{\mathbf{v}} \bullet \overrightarrow{\mathbf{u}}$.

(b) Find $\overrightarrow{\mathbf{u}} \times \overrightarrow{\mathbf{v}}$ and $\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{u}}$.

(c) Find the two unit vectors parallel to $\overrightarrow{\mathbf{u}} \times \overrightarrow{\mathbf{v}}$.

(d) Find the vector (and scalar) projection of $\overrightarrow{\mathbf{u}}$ onto $\overrightarrow{\mathbf{v}}$.

(e) Find the vector (and scalar) projection of $\overrightarrow{\mathbf{v}}$ onto $\overrightarrow{\mathbf{u}}$.

(h) Is the cross product of two unit vectors always a unit vector?

**Example 2:** Let $\overrightarrow{\mathbf{u}} = \langle 4, 3, 0\rangle$ and $\overrightarrow{\mathbf{v}} = \langle 5, 5, 5\rangle$ and $\overrightarrow{\mathbf{w}} = \langle 2, 3, c\rangle$.

(a) Find the angle between $\overrightarrow{\mathbf{u}}$ and $\overrightarrow{\mathbf{v}}$.

(b) Find the volume of the parallelepiped determined by $\overrightarrow{\mathbf{u}}, \overrightarrow{\mathbf{v}}, \overrightarrow{\mathbf{w}}$.

(c) Find $c$ such that vectors $\overrightarrow{\mathbf{u}}, \overrightarrow{\mathbf{v}}, \overrightarrow{\mathbf{w}}$ are coplanar.

(d) Find $c$ such that $\overrightarrow{\mathbf{u}} \times \overrightarrow{\mathbf{v}}$ is orthogonal to $\overrightarrow{\mathbf{w}}$.

**Problem 2:** (Section 12.5) Lines, planes, angle between two planes, distance from a point to a plane, and many other topics.

**Example:** Consider the plane $\mathbf{P} : x + y + z = 3$ and the line $L_1$: $\overrightarrow{\mathbf{r}}(t) = \langle t-1, 2t, -t+1\rangle$. (i) Determine whether the plane and the line intersect or parallel. (ii) If intersect, find the point of intersection. If parallel, find the distance between them. (iii) Find a line $L_2$ on the plane $\mathbf{P}$ such that $L_1$ and $L_2$ are intersect and orthogonal. (iv) Consider point $Q(1, 2, 3)$ which is not on the plane $\mathbf{P}$. Find the point $R$ on the plane $\mathbf{P}$ such that the vector $\overrightarrow{RQ}$ is orthogonal to the plane.

**Problem 3:** (Section 12.6) Cylinders and and quadric surfaces in $\mathbf{R}^3$.

**Example 1:** (a) A surface consists of all points $P$ such that the distance from $P$ to $(1, 0, -1)$ is twice the distance from $P$ to the plane $y = 1$. Find an equation for this surface and identify it.

(b) Describe the curve of intersections of the surface $P$ and the $xz$-plane.

**Example 2:** Consider the surface

$$x^2 - 2x + y^2 - 4y + cz^2 - 8z = 0$$

where $c$ is a constant. Determine the interval for $c$ such that the surface is a hyperboloid of two sheets.

**Problem 4:** (Section 10.3) Polar curves.

**Example:** (a) Let $C_1$ be the polar curve determined by the polar equation

$$r = -2\cos(\theta).$$

Convert this polar equation to a Cartesian equation only involving $x$ and $y$.
(b) Let $C_2$ be the curve determined by the equation

$$(x + 1)^2 + (y + 1)^2 = 2.$$

Convert the above Cartesian equation to a polar equation only involving $r$ and $\theta$.
(c) Find the intersection points of curves $C_1$ and $C_2$, use Cartesian $(x, y)$-coordinates.

**Note that problems in the exam may not be in this order and that practice problems may not occur in the exam.**

**Please review all homework problems.**

Old Exam1 can be found at
https://sites.math.washington.edu/~m126/midterms/midterm1.php