Final Exam–Math 126 A/B, Fall 2012

Final Exam will be given on Saturday, Dec. 8, from 1:30-4:20pm. It will cover Taylor Notes, Ch. 10, 12, 13, 14 and 15.

Suggestions: Use old final exams as practice tests.

Some rules

1. You may use a simple scientific calculator, but not graphing calculators.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work (except for True/False questions). You will not receive any partial credit unless all work is clearly shown.
4. Unless otherwise stated, always give your answers in exact form. For example, $3\pi$, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are 9 questions in the exam. Each question contains several parts.
6. Different topics could be combined in one question in the final exam.
7. Place a box around your final answer to each question.

Review topics

1: True-False questions.
2: Taylor series and operations with Taylor Series, $\Sigma$ notation and coefficients of Taylor series. Interval of convergence and radius of convergence.
3: Taylor polynomials, approximation, error bounds, finding $M_{n+1}$, $n$, $I$ etc. Connection between Taylor polynomials and Taylor series.
4: Mass, moments and the center of Mass.
5: Double integrals, volume/area, average value, double integrals in polar coordinates, sketching the region.
6: Functions of several variables, level curves (contour maps) and partial derivatives.
7: Tangent plane and linear approximation.
8: Critical points, maximum/minimum values, second derivative test, extreme values on boundary.
9: Story problem (optimization problem).
10: $T N B$ system, curvature, arclength, curve sketching. Normal plane and osculating plane.
11: Vector functions and curves in 3D, derivatives and integrals of vector functions, velocity and acceleration, tangential and normal components of acceleration.

12: Equations of lines and planes in 3D. Intersection, angle of intersection, distance from a point to a plane. Parallel, skew, intersect?

13: Cylinders and quadric surfaces in 3D.

14: Scalar projection, vector projection, triple product, direction angles, angle formulas, area and volume formulas.

15: Vector, length, angle, dot product, cross product.

16: Parametric curves in 2D, tangent, area, arclength, polar curves, tangents to polar curves.

Practice problems for Taylor polynomials and Taylor Series

1. Consider the function

   \[ f(x) = \frac{x}{1 + 7x^2} + \int_0^x e^{t^2} dt. \]

   (a) Find the Taylor series for \( f(x) \) based at \( b = 0 \). Write your answer using one sigma sign. (b) Give the open interval of convergence for the Taylor series in part (a).

2. Consider the function \( f(x) = \frac{3}{7x^4} - \frac{\sin x}{x} \). (a) Find the Taylor series for \( f(x) \) based at \( b = 0 \). (b) Find the first four nonzero terms in part (a). (c) Give the open interval of convergence for the Taylor series in part (a). (d) Find the 5th Taylor polynomial for \( f(x) \) based at \( b = 0 \).

3. Consider the function

   \[ f(x) = x^2 \sin x + 3 \ln x \]

   (a) Find the second Taylor polynomial \( T_2(x) \) for \( f(x) \) based at \( b = 1 \). (b) Use the Taylor inequality to bound the error on the interval \( I = [0.8, 1.2] \).

4. Consider the function

   \[ f(x) = \sin(x^2 - 1) \]

   (a) Find the second Taylor polynomial \( T_2(x) \) for \( f(x) \) based at \( b = 1 \). (b) Use the second Taylor polynomial \( T_2(x) \) to approximate \( f(1.01) \). (c) Use the Taylor inequality to find an interval \( J \) containing \( b \) so that the error bound is at most 0.001.