Midterm Two–Math 125 A/B, Spring 2015

Midterm Two will be given on Thursday, May 21, 2015 in quiz section. It will cover Sections 6.4-8.1.

**Some basic rules**

1. **No** calculators are allowed in exams.
2. You may use one page of two-sided handwritten notes (standard 8.5 × 11 sheet).
3. Make sure to show your work. You will not receive any partial credit unless all work is clearly shown.
4. Unless otherwise stated, always give your answers in exact form. For example, $3\pi$, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are five questions in this exam. Each question contains several parts.
6. You can use integral formulas on page 495.
7. No make-up exam.

**Practice problems**

(1) **Work $6.4$.**

Ex1: A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

Ex2: A circular swimming pool has a diameter of 20 ft, the sides are 10 ft high, and the depth of the water is 8 ft. How much work is required to pump all of the water out over the side? (Use the fact that water weighs 62.5 lb/ft³.)

(2,3) **Techniques of integration, Chapter 7.**

Example: Evaluate the following integrals.

(a) $\int e^{x^{1/3}}dx$.
(b) $\int_0^{\pi^2} \sqrt{x} \sin(\sqrt{x})dx$.
(c) $\int \frac{2-3x}{\sqrt{x^2-4x+3}}dx$.
(d) $\int \frac{2x^4-x^2+2}{x^2+2x^2+2x}dx$.
(e) $\int \cos^4 t \sin^2 t \, dt$. 
(f) \( \int \frac{x^2}{(3+4x-4x^2)^{3/2}} \, dx. \)
(g) \( \int_0^1 \frac{1}{(x^2+1)^2} \, dx. \)
(i) \( \int \tan^3 \theta \, d\theta. \)
(j) \( \int \sec^3 \theta \sin^2 \theta \cos \theta \csc^2 \theta \cot^3 \theta \, d\theta. \)

* Two definite integrals and two indefinite integrals.

See pp499-450, pp530-531 for more practice problems.

(4) Approximate integration and arc length, §7.7 and §8.1.

(4a) Find the length of the curve
\( y = \arcsin(\sqrt{x}) - \sqrt{x - x^2} \)
for \( 0 \leq x \leq \frac{1}{2}. \)

(4b) Find the length of the curve
\( y = \frac{x^3}{3} + \frac{1}{4x} \)
for \( 1 \leq x \leq 2. \)

(4c) Use Simpson’s rule to approximate \( \int_1^2 \sqrt{1 + x^4} \, dx \) with \( n = 4. \)

* We might combine (4a/b) with (4c).

(5) Average value, improper integrals § 6.5 and §7.8.

(5a) Find the average of the function \( y = e^{\sin x} \cos x \) on interval \([0, \pi/2].\)

(5b) Determine whether the following integrals are convergent or divergent
\( \int_1^\infty \frac{x+1}{\sqrt{x^4-x}} \, dx, \quad \text{and} \quad \int_0^1 x^p \ln x \, dx \) where \( p > 0. \)

* You also need to know comparison test.