Final Exam–Math 124 C/D, Spring 2007
Page 1

The final exam will be given on Saturday, June 2 at KNE 110 from 1:30-4:20. It will cover Chapters 1, 2, 3 and 4. You may use a simple scientific calculator and one page of two-sided handwritten notes (standard 8.5 × 11 sheet). Graphing calculators are not allowed on exams.

I Limits: (a) Easy limits. (b) Limits with \( \infty \). (c) L’Hospital rule.

Example: Find the following limits.

\[
\begin{align*}
(1) \lim_{x \to \frac{\pi}{2}} & \frac{\sin x - 1}{\cos x}, \\
(2) \lim_{x \to 1} & \frac{4x^3 + 3x - 7}{x^3 - 1}, \\
(3) \lim_{t \to \infty} & \frac{e^t + \sin t}{t + 1}, \\
(4) \lim_{t \to \infty} & \sqrt{\frac{t^4 + t^3 - 1}{9t^4 + 10001}}. \\
(5) \lim_{x \to \frac{\pi}{2}^+} & \frac{\tan x - 1}{\sqrt{\cos x}}, \\
(6) \lim_{x \to 1^-} & 4x^3 + 3x - 7, \\
(7) \lim_{t \to 1^+} & \frac{\cos t}{t - 1}, \\
(8) \lim_{t \to 0^-} & t^4 - t + 10001.
\end{align*}
\]

II Derivatives: (a) Using limit/definition to find the derivative. (b) Basic rules. (c) Implicit differentiation. (d) Logarithmic differentiation.

Example 1: Find the derivative of the function \( f(x) = x + \sqrt{x} \) using the definition of the derivative. Do not use any differentiation formula.

Example 2: Find the derivatives of the following functions

\[ y = 5x^7 - \sin(3x) + e^x + \ln(x + 1), \]
\[ f(r) = (1 + \sqrt{r})^3(1 - \frac{1}{\sqrt{r}})^4, \]
\[ g(t) = (\ln t)^{\ln t + t}. \]

\[ y = \sqrt{x} - \frac{1}{\sqrt{3x - 5}}, \quad g(w) = e^w \sin(e^{3w}), \quad h(t) = \frac{\tan(t^2)}{\cos^3(2t)}, \quad f(x) = (x^2 + 1)^{x^2 + 1}. \]

Example 3: Consider the curve given by the equation \( x^2 + xy + y^2 = x^2y^2 \). Compute the equation of the tangent line to the curve at the point \((\sqrt{3}, \sqrt{3})\).

III Curve sketching: (a) Critical points. (b) Increasing/decreasing. (c) Concavity and inflection points. (d) Curve sketching \((x,y)-intercepts, asymptotes, etc)\)

Example 1: Let \( f(x) = x^4 - x^2 \).

(a) Calculate the intervals in which \( f(x) \) is increasing and decreasing.

(b) Find the critical points for this function. For each, determine whether it is a local minimum, local maximum or neither.

(c) Identify the intervals on which \( f(x) \) is concave up and concave down.

(d) Use the curve sketching procedure to carefully and clearly graph \( f(x) \). (Include BOTH coordinates of all local minima and maxima, inflection points, \( x \)-intercept(s) and \( y \)-intercept(s).)

Example 2: Let \( f(x) = xe^{-x^2} \). (a,b,c,d) as above.