**Brownian penalizations related to local time and excursion lengths**

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Let $P_0$ denote the Wiener measure defined on the canonical space $\big(\Omega = \mathcal{C}(\mathbb{R}_+, \mathbb{R}),\ (X_t)_{t\geq 0},\ (\mathcal{F}_t)_{t\geq 0}\big)$.

1) Let $(L_t^0)$ be the local time at 0 and $(\Gamma_t)$ the process : $\Gamma_t = \varphi(L_t^0)$, $t \geq 0$, where $\varphi : \mathbb{R}_+ \to ]0, +\infty[$ is a Borel function such that $\displaystyle\int_0^\infty \varphi(x)dx = 1$.

We prove the following penalization result :

$$\lim_{t\to\infty} \frac{E_0\big[1_{\Gamma_u}\Gamma_t\big]}{E_0[\Gamma_t]} := Q_0^\Gamma(\Gamma_u), \quad \forall\,\Gamma_u \in \mathcal{F}_u \text{ and } \ u \geq 0, \tag{0.1}$$

where $Q_0^\Gamma(\Gamma_u) = E_0\big[1_{\Gamma_u} M_u^\varphi\big]$, and $(M_u^\varphi)$ is the $P_0$-martingale :

$$M_u^\varphi = \varphi(L_u^0)|X_u| + 1 - \int_0^{L_t^0} \varphi(z)dz.$$

We determine the law of $(X_t)$ under the p.m. $Q_0^\Gamma$ defined on $\big(\Omega, \mathcal{F}_\infty\big)$ by the relation above. We prove in particular that $Q_0^\Gamma(L_\infty^0 < \infty) = 1$.

It can be shown that, under $Q_0^\Gamma$, the process $(X_t)$ satisfies the Pitman property : $(R_t = |X_t| + L_t^0)_{t\geq 0}$ is in its own filtration, a three dimensional Bessel process started at 0. Moreover there exists a version of Ray-Knight theorem.

We also investigate the case where $(X_t)$ is a $d$-dimensional Bessel process started at 0, with $0 < d < 2$.

2) We consider penalization related to $\Gamma_t = 1_{\{\Sigma_t \leq a\}}$, $t \geq 0$ where $a$ is a positive constant and $\Sigma_t$ is the length of the longest complete excursion of $(X_t)$ before time $t$. We prove that a penalization result of the type (0.1) holds and we determine the associated martingale and the law of the canonical process under $Q^\Gamma$.