

Rigidity of entire self-shrinking solutions to curvature flows

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Abstract. We show (a) that any entire graphic self-shrinking solution to the Lagrangian mean curvature flow in \mathbb{C}^n with the Euclidean metric is flat; (b) that any space-like entire graphic self-shrinking solution to the Lagrangian mean curvature flow in \mathbb{C}^n with the pseudo-Euclidean metric is flat if the Hessian of the potential is bounded below quadratically; and (c) the Hermitian counterpart of (b) for the Kähler Ricci flow.

1. Introduction

Self-similar solutions to curvature flows play an important role in understanding the general behavior of the flow and the types of singularities that can develop. For mean curvature flow, self-shrinking solutions arise naturally at a type-I singularity from Huisken's monotonicity formula [7]. More precisely, these are ancient families of immersions $F(x, t) : \Sigma \times (-\infty, 0) \rightarrow \mathbb{R}^N$ of some manifold Σ into \mathbb{R}^N which solve the mean curvature flow equation

$$(1) \quad \left(\frac{d}{dt} F \right)^\perp = H$$

simply by scaling $F(\Sigma, t) = \sqrt{-t}F(\Sigma, -1)$. Here $\left(\frac{d}{dt} F \right)^\perp$ is the normal component of the vector $\frac{d}{dt} F$ and H is the mean curvature of $F(\Sigma, t)$. It follows that $F(x, -1)$ satisfies the equation

$$(2) \quad H + \frac{1}{2} F^\perp = 0.$$

Conversely, if an embedding F satisfies (2) then the corresponding solution to the mean curvature flow will be a self-shrinking solution. If a Lagrangian graph $\{(x, Du(x)) : x \in \mathbb{R}^n\}$

The first two authors are partially supported by NSERC, and the third author is partially supported by NSF.

in \mathbb{R}^{2n} satisfies (2), then up to an additive constant the potential function u solves

$$(3) \quad \arctan \lambda_1(x) + \cdots + \arctan \lambda_n(x) = \frac{1}{2}x \cdot Du(x) - u(x)$$

where $\lambda_1(x), \dots, \lambda_n(x)$ are the eigenvalues of the Hessian D^2u of u at $x \in \mathbb{R}^n$. The first main result in this note is the following Bernstein type rigidity for entire self-shrinking solutions for Lagrangian mean curvature flow:

Theorem 1.1. *If $u(x)$ is an entire smooth solution to equation (3) in \mathbb{R}^n , then $u(x)$ is the quadratic polynomial $u(0) + \frac{1}{2}\langle D^2u(0)x, x \rangle$.*

When $\mathbb{R}^{2n} = \{(x, y) : x, y \in \mathbb{R}^n\}$ is equipped with the indefinite metric $\sum_{i=1}^n dx_i dy_i$, if a space-like gradient graph $\{(x, Du(x)) : x \in \mathbb{R}^n\}$ satisfies (2) then up to an additive constant the potential u is convex and satisfies the elliptic equation

$$(4) \quad \ln \det D^2u(x) = \frac{1}{2}x \cdot Du(x) - u(x).$$

We have

Theorem 1.2. *If $u(x)$ is an entire smooth convex solution to (4) in \mathbb{R}^n , then $u(x)$ is the quadratic polynomial $u(0) + \frac{1}{2}\langle D^2u(0)x, x \rangle$, provided either*

$$(i) \quad D^2u(x) \geq \frac{2(n-1+\delta)}{|x|^2}I \text{ for any } \delta > 0 \text{ as } |x| \rightarrow \infty$$

or

(ii) u is radially symmetric.

Finally, we also consider the Hermitian analog of (4) and Theorem 1.2. Namely we consider real valued functions $v : \mathbb{C}^m \rightarrow \mathbb{R}$ satisfying

$$(5) \quad \ln \det \partial\bar{\partial}v(x) = \frac{1}{2}x \cdot Dv(x) - v(x)$$

on \mathbb{C}^m . This is closely related to the Kähler Ricci flow as we describe in Section 4. We prove

Theorem 1.3. *If $v(x)$ is an entire smooth pluri-subharmonic solution to (5) in \mathbb{C}^m , then $v(x)$ is the quadratic polynomial $v(0) + \frac{1}{2}\langle D^2v(0)x, x \rangle_{\mathbb{R}^{2m}}$, provided either*

$$(i) \quad \partial\bar{\partial}v(x) \geq \frac{2m-1+\delta}{2|x|^2}I \text{ for any } \delta > 0 \text{ as } |x| \rightarrow \infty$$

or

(ii) v is radially symmetric.

By using existence [1] and uniqueness [3] results for the Lagrangian mean curvature flow, the rigidity of self-expanding, self-shrinking and translating solutions for the Lagrangian mean curvature flow was studied in [2] when the Hessian of the potential function is uniformly bounded between -1 and 1 . The same rigidity for self-shrinking and translating solutions with arbitrarily bounded Hessian was derived from a Liouville type property for ancient solutions to parabolic equations in [8] (for self-shrinking solutions, a special case of [8] was treated recently in [6]). Theorem 1.1 improves the previous results on self-shrinking solutions by dropping the assumption on Hessian completely. For the pseudo-Euclidean case, under a similar assumption on the Hessian as in Theorem 1.2, namely quadratically decaying lower bound, the Bernstein type result was obtained in [6]. However, our method is completely different and much simpler, and further, gives a slightly sharper constant in the assumption on the Hessian (when scaling to the same equation (4)). Recently, it was shown in [10] that any entire graphic hypersurface solution to (2) must be flat, hence generalizing an earlier result in [4].

A key ingredient in our arguments, for each of the three cases above, is that a natural geometric quantity, involving second order derivatives of the potential function, obeys a second order elliptic equation with an ‘‘amplifying force’’. We then construct a barrier function to show that the quantity is constant via the maximum principle. Finally, the homogeneity of the lower order terms in the equations implies the potentials are quadratic polynomials.

2. Proof of Theorem 1.1

We point out that if u satisfies (3) then $v(x, t) = -tu\left(\frac{x}{\sqrt{-t}}\right)$ satisfies

$$(6) \quad \frac{\partial v}{\partial t} = -\sqrt{-1} \log \frac{\det(I + \sqrt{-1}D^2v)}{\sqrt{\det(I + D^2vD^2v)}}$$

on $\mathbb{R}^n \times (-\infty, 0)$ and the family of embeddings $F(x, t) = (x, Dv(x, t))$ from \mathbb{R}^n into \mathbb{R}^{2n} solves the mean curvature flow (1) (cf. [9]). While this connection is our main motivation to study (3), we will not use (6) explicitly in our following proof of Theorem 1.1.

Let $z_j = x_j + \sqrt{-1}y_j$ be the standard complex coordinates on $\mathbb{C}^n = \mathbb{R}^{2n}$. The phase function Θ on a Lagrangian submanifold Σ^n is defined by

$$dz_1 \wedge \cdots \wedge dz_n|_{\Sigma^n} = e^{\sqrt{-1}\Theta} d\mu_{\Sigma^n}$$

and when Σ^n is a Lagrangian graph $\{(x, Du(x)) : x \in \mathbb{R}^n\}$ in \mathbb{C}^n , Θ takes the form

$$(7) \quad \Theta = \arctan \lambda_1 + \cdots + \arctan \lambda_n$$

(cf. [5]). For simplicity, for a function f , we denote $\frac{\partial^k f}{\partial x_{i_1} \cdots \partial x_{i_k}}$ by $f_{i_1 \cdots i_k}$ for $k = i_1 + \cdots + i_k$.
Let

$$\begin{aligned} A &= (A_{ij}) = I + \sqrt{-1}D^2u, & (A^{ij}) &= A^{-1}, \\ B &= (B_{ij}) = I + D^2uD^2u, & (B^{ij}) &= B^{-1}. \end{aligned}$$

Observe that

$$(8) \quad B = (I - \sqrt{-1}D^2u)A \quad \text{and} \quad A^{-1} = B^{-1}(I - \sqrt{-1}D^2u)$$

and

$$(9) \quad \Theta = -\sqrt{-1} \log \frac{\det A}{\sqrt{\det B}}.$$

Then we can differentiate (9) as follows:

$$\begin{aligned} (10) \quad \Theta_k &= -\sqrt{-1} \sum_{i,j=1}^n \left(A^{ij} A_{ij,k} - \frac{1}{2} B^{ij} B_{ij,k} \right) \\ &= -\sqrt{-1} \sum_{i,j,l=1}^n \left(B^{il} (\delta_{lj} - \sqrt{-1}u_{lj}) \cdot \sqrt{-1}u_{ijk} - \frac{1}{2} B^{ij} \cdot 2u_{il}u_{ljk} \right) \\ &= \sum_{i,j=1}^n B^{ij} u_{ijk} - \sqrt{-1} \sum_{i,j,l=1}^n (B^{il} u_{lj} u_{ijk} - B^{ij} u_{il} u_{ljk}) \\ &= \sum_{i,j=1}^n B^{ij} u_{ijk}, \end{aligned}$$

where we used the symmetry of B .

Differentiating equation (3), and using (7) we have

$$(11) \quad \Theta_i = -\frac{1}{2}u_i + \frac{1}{2}x \cdot Du_i$$

and

$$(12) \quad \Theta_{ij} = \frac{1}{2}x \cdot Du_{ij} = \frac{1}{2} \sum_{k=1}^n x_k u_{ijk}.$$

Note that B is just the induced metric g of Σ^n in \mathbb{C}^n with the Euclidean metric. It follows from (12) and (10) that Θ satisfies the following elliptic equation of non-divergence form:

$$(13) \quad \sum_{i,j=1}^n g^{ij} \Theta_{ij}(x) - \frac{1}{2}x \cdot D\Theta(x) = 0$$

with the ‘‘amplifying force’’ $\frac{1}{2}x \cdot D\Theta(x)$.

Next, we construct a barrier to show Θ attains its global maximum at an interior point. Take a radially symmetric function

$$(14) \quad w(r) = \varepsilon r^{1+\delta} + \max_{\partial B_{r_0}} \{\Theta\}$$

where ε is any positive constant and B_{r_0} is the ball in \mathbb{R}^n centered at the origin with radius $r_0 = \sqrt{2(n-1+\delta)}$. For $|x| = r \geq r_0$, we have

$$(15) \quad w_{rr} + \frac{n-1}{r}w_r - \frac{r}{2}w_r \leq 0$$

where

$$w_r = \varepsilon(1+\delta)r^\delta > 0,$$

$$w_{rr} = \frac{\delta}{r}w_r > 0.$$

Also for each $x \neq 0$, we can find a rotation of coordinates in \mathbb{R}^n so that we have the following similarity of matrices:

$$D^2w(r) \sim \begin{pmatrix} w_{rr} & & & \\ & \frac{w_r}{r} & & \\ & & \dots & \\ & & & \frac{w_r}{r} \end{pmatrix} \geq 0.$$

Observe that

$$(16) \quad g^{-1} = (I + D^2uD^2u)^{-1} \leq I.$$

Thus we have

$$(17) \quad \text{tr}(g^{-1}D^2w) \leq \text{tr}(ID^2w).$$

Hence, when $|x| \geq r_0$ we have

$$(18) \quad \sum_{i,j=1}^n g^{ij}w_{ij} - \frac{1}{2}x \cdot Dw \leq \Delta w - \frac{1}{2}x \cdot Dw \leq 0,$$

where Δ is the Euclidean Laplacian on \mathbb{R}^n and we have used (15) in the last inequality.

So far we have

$$(19) \quad \sum_{i,j=1}^n g^{ij}w_{ij} - \frac{1}{2}x \cdot Dw \leq \sum_{i,j=1}^n g^{ij}\Theta_{ij} - \frac{1}{2}x \cdot D\Theta \quad \text{if } r_0 \leq |x| < \infty$$

with comparison along the boundaries:

$$(20) \quad w = \varepsilon r_0^{1+\delta} + \max_{\partial B_{r_0}}\{\Theta\} \geq \Theta \quad \text{on } \partial B_{r_0}$$

and

$$(21) \quad w(|x|) > \Theta(x) \quad \text{when } |x| \rightarrow \infty,$$

since Θ is bounded while $w(|x|) \rightarrow \infty$ as $|x| \rightarrow \infty$. By the weak maximum principle, we get

$$(22) \quad \varepsilon|x|^{1+\delta} + \max_{\partial B_{r_0}}\{\Theta\} = w(|x|) \geq \Theta(x)$$

for all $|x| \geq r_0$. By letting ε go to zero, we then conclude that Θ achieves its global maximum on \mathbb{R}^n in the closure of the ball B_{r_0} . Applying the strong maximum principle to (13), we immediately see that Θ is a constant.

Now from (12), for any i, j we have

$$(23) \quad x \cdot Du_{ij} = 0.$$

Euler's homogeneous function theorem asserts that u_{ij} is homogeneous of degree 0. Moreover, the function u_{ij} is smooth at the origin, therefore u_{ij} is constant. It follows from (11) that u is the quadratic polynomial in the claimed form. This completes the proof of Theorem 1.1.

Remark. Denote $\tilde{F}(x, t) = (x, Dv(x, t))$. Then by recalling $v(x, t) = \sqrt{-tu} \left(\frac{x}{\sqrt{-t}} \right)$ we have that (13) is equivalent to

$$(24) \quad \frac{\partial \Theta(\tilde{F})}{\partial t} = \sum_{i,j=1}^n g^{ij}(\tilde{F}) \Theta(\tilde{F})_{ij}.$$

It is known that if $F(x, t)$ satisfies the mean curvature flow equation $F_t = H(F)$ and F is Lagrangian, then its phase function Θ satisfies

$$\frac{\partial \Theta(F)}{\partial t} = \Delta_g \Theta(F)$$

where Δ_g is the Laplace operator of the induced metric g on the time slice $F(\cdot, t)$. The non-divergence structure of (24) is due to the fact that \tilde{F} satisfies the mean curvature flow equation up to tangential diffeomorphisms.

3. Proof of Theorem 1.2

We note that if u satisfies (4) then $v(x, t) = -tu \left(\frac{x}{\sqrt{-t}} \right)$ verifies

$$(25) \quad \frac{\partial v}{\partial t} = \ln \det D^2 u(x)$$

on $\mathbb{R}^n \times (-\infty, 0)$ and the family of embeddings $F(x, t) = (x, Dv(x, t))$ from \mathbb{R}^n into \mathbb{R}^{2n} solves the mean curvature flow (1) with respect to the pseudo-Euclidean background metric $ds^2 = \sum_{i=1}^n dx^i dy^i$ on \mathbb{R}^{2n} (cf. [6]). Again, while this connection is our main motivation to study (4), we will not use (25) explicitly in our following proof of Theorem 1.2.

From (4), we see that $D^2u > 0$. Set $\Psi = \ln \det D^2u$. We have

$$\Psi_i = \sum_{k,l=1}^n u^{kl} u_{kli} = \sum_{k,l=1}^n g^{kl} u_{kli}$$

where g^{-1} is the inverse of the induced metric g of the graph $(x, Du(x))$ in \mathbb{R}^{2n} with the pseudo-Euclidean metric above. On the other hand, by differentiating equation (4) twice we obtain

$$\Psi_{ij}(x) = \frac{1}{2} x \cdot Du_{ij}(x)$$

and hence,

$$(26) \quad \sum_{i,j=1}^n g^{ij} \Psi_{ij}(x) - \frac{1}{2} x \cdot D\Psi(x) = 0.$$

Next, as in the previous section, for any $\varepsilon > 0$ we take a radially symmetric function w defined by

$$(27) \quad w(r) = \varepsilon r^{1+\delta} + \max_{\partial B_1} \{-\Psi\}.$$

It is clear that for r positive

$$(28) \quad \begin{aligned} w_r &= \varepsilon(1+\delta)r^\delta > 0, \\ w_{rr} &= \frac{\delta}{r} w_r > 0, \\ \frac{r^2}{2(n-1+\delta)} \Delta w - \frac{r}{2} w_r &= 0, \end{aligned}$$

where Δ is the Euclidean Laplacian on \mathbb{R}^n and

$$D^2w > 0.$$

By assumption (i) in Theorem 1.2, we have

$$(29) \quad g^{-1} = (D^2u)^{-1} \leq \frac{|x|^2}{2(n-1) + 2\delta} I.$$

Here we assume $D^2u \geq \frac{2(n-1+\delta)}{|x|^2}I$ for $|x| > 1$ instead of $|x|$ being greater than a large number as in the assumption for simplicity. Otherwise, we just replace 1 by the large number, and our arguments go through as well. Thus

$$(30) \quad \operatorname{tr}(g^{-1}D^2w) \leq \frac{r^2}{2(n-1)+2\delta} \operatorname{tr}(ID^2w)$$

and it follows that

$$(31) \quad \sum_{i,j=1}^n g^{ij}w_{ij} - \frac{1}{2}x \cdot Dw \leq \frac{r^2}{2(n-1)+2\delta} \Delta w - \frac{1}{2}rw_r = 0$$

where we have used (28) to conclude the last equality.

Thus far, we have

$$(32) \quad \sum_{i,j=1}^n g^{ij}w_{ij} - \frac{1}{2}x \cdot Dw \leq \sum_{i,j=1}^n g^{ij}(-\Psi)_{ij} - \frac{1}{2}x \cdot D(-\Psi).$$

Also, we have that along the boundaries

$$w(|x|) = \varepsilon + \max_{\partial B_1} \{-\Psi\} \geq -\Psi(x) \quad \text{on } \partial B_1$$

and

$$w(|x|) > -\Psi(x) \quad \text{as } |x| \rightarrow \infty$$

by the assumption on D^2u in (i). The weak maximum principle then implies

$$\varepsilon|x|^{1+\delta} + \max_{\partial B_1} \{-\Psi\} = w(x) \geq -\Psi(x) \quad \text{for any } x \in \mathbb{R}^n \setminus B_1.$$

Letting $\varepsilon \rightarrow 0$, we obtain

$$\max_{\partial B_1} \{-\Psi\} \geq -\Psi(x) \quad \text{for any } x \in \mathbb{R}^n \setminus B_1.$$

So Ψ attains its global minimum on \mathbb{R}^n in the closure of B_1 . Hence Ψ is a constant by applying the strong maximum principle to equation (26). Now as in the proof of Theorem 1.1, we conclude that u must be the quadratic polynomial in the desired form by differentiating equation (4). Part (i) of Theorem 1.2 is proved.

If instead of assumption (i) in Theorem 1.2 we assume that u is radially symmetric, then Ψ is also radially symmetric and depends only on $|x|$. It follows that Ψ must then attain either a local maximum or a local minimum over any open ball B in \mathbb{R}^n . The strong maximum principle then implies Ψ is constant in B , and hence in \mathbb{R}^n . As before, we conclude that u is quadratic. Part (ii) of Theorem 1.2 is proved.

4. Proof of Theorem 1.3

Equation (5) is in fact closely related to the Kähler Ricci flow equation

$$(33) \quad \frac{\partial g_{i\bar{j}}}{\partial t} = -R_{i\bar{j}}.$$

Indeed, if v is a strictly-plurisubharmonic solution to (5), then it follows that $u(x, t) = -tv \left(\frac{x}{\sqrt{-t}} \right)$ solves the parabolic complex Monge–Ampère equation

$$(34) \quad \frac{\partial u}{\partial t} = \ln \det(\partial\bar{\partial}u)$$

and the Kähler metrics $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} u$ will evolve according to (33). Although $g_{i\bar{j}}$ in general is not a gradient shrinking Kähler Ricci soliton, Peng Lu pointed out in an oral communication that it is a shrinking Kähler Ricci soliton. In particular, for every $t \in (-\infty, 0)$ the biholomorphism $z \rightarrow \frac{z}{\sqrt{-t}}$ is an isometry from $(\mathbb{C}^m, g_{i\bar{j}}(t))$ to $(\mathbb{C}^m, g_{i\bar{j}}(-1))$.

Conversely, a self-similar solution of (34) $u(x, t) = -tv \left(\frac{x}{\sqrt{-t}} \right)$ leads to

$$(35) \quad \ln \partial\bar{\partial}v \left(\frac{x}{\sqrt{-t}} \right) = \frac{1}{2} \frac{x}{\sqrt{-t}} \cdot Dv \left(\frac{x}{\sqrt{-t}} \right) - v \left(\frac{x}{\sqrt{-t}} \right).$$

For a solution v to (5), introduce the notations

$$\begin{aligned} \Phi(x) &= \ln \det(\partial\bar{\partial}v(x)), \\ g^{i\bar{j}}(x) &= v^{i\bar{j}}(x). \end{aligned}$$

In the following, we verify

$$(36) \quad \sum_{i,j=1}^m g^{i\bar{j}} \Phi_{i\bar{j}}(x) - \frac{1}{2} x \cdot D\Phi(x) = 0.$$

We first calculate

$$\Phi_i = \sum_{k,l=1}^m g^{k\bar{l}} v_{k\bar{l}i} \quad \text{and} \quad \Phi_{\bar{j}} = \sum_{k,l=1}^m g^{k\bar{l}} v_{k\bar{l}\bar{j}}$$

therefore

$$x \cdot D\Phi = \sum_{k=1}^m (z_k \partial_{z_k} \Phi + \bar{z}_k \partial_{\bar{z}_k} \Phi) = \sum_{i,j,k=1}^m g^{i\bar{j}} (z_k v_{k\bar{l}i} + \bar{z}_k v_{k\bar{l}\bar{j}}).$$

On the other hand, from (35) we have

$$\begin{aligned} \sum_{i,j=1}^m g^{i\bar{j}} \Phi_{i\bar{j}} &= \sum_{i,j=1}^m g^{i\bar{j}} \left(\frac{1}{2} x \cdot Dv - v \right)_{i\bar{j}} \\ &= \sum_{i,j=1}^m g^{i\bar{j}} \left(\frac{1}{2} \sum_{k=1}^m (z_k \partial_{z_k} v + \bar{z}_k \partial_{\bar{z}_k} v) - v \right)_{i\bar{j}} \\ &= \frac{1}{2} \sum_{i,j,k=1}^m g^{i\bar{j}} (z_k v_{k i\bar{j}} + \bar{z}_k v_{\bar{k} i\bar{j}}), \end{aligned}$$

from which we conclude that equation (36) holds.

Now take the radial barrier function

$$w(r) = \varepsilon r^{1+\delta} + \max_{\partial B_1} \{-\Phi\}$$

as in the previous section. Then we have

$$\frac{|x|^2}{2(2m-1+\delta)} \Delta w(|x|) - \frac{1}{2} x \cdot Dw(|x|) = 0.$$

Moreover, the assumption on the complex Hessian $\partial\bar{\partial}v$ implies

$$g^{-1} \leq \frac{2r^2}{2m-1+\delta} I.$$

Here we assume $\partial\bar{\partial}v \geq \frac{(2m-1+\delta)}{2|x|^2} I$ for $|x| > 1$ instead of $|x|$ being greater than a large number as in the assumption for simplicity. Otherwise, we just replace 1 by the large number, and our arguments go through as well. Note that $\partial\bar{\partial}w > 0$, it follows that

$$\sum_{i,j=1}^m g^{i\bar{j}} w_{i\bar{j}} - \frac{1}{2} x \cdot Dw \leq \frac{2|x|^2}{(2m-1+\delta)} \frac{1}{4} \Delta w - \frac{1}{2} x \cdot Dw = 0.$$

Then as in the proof of Theorem 1.2, the weak maximum principle implies the smooth function $-\Phi$ achieves its global maximum in the closure of B_1 and the strong maximum principle asserts Φ is constant. In turn, we conclude that v is the quadratic polynomial in the claimed form by differentiating equation (5). Part (i) of Theorem 1.3 is proved.

The radially symmetric case part (ii) follows exactly as in the proof of Theorem 1.2.

Acknowledgment. We are grateful to Peng Lu for the discussion about Kähler Ricci solitons.

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Eingegangen 27. März 2010, in revidierter Fassung 27. August 2010