Linearity of Homogeneous Order-One Solutions to Elliptic Equations in Dimension Three

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1 Introduction

In this note we prove that any homogeneous order 1 solution to nondivergence elliptic equations in \( \mathbb{R}^3 \) must be linear. Consider the general equations in \( \mathbb{R}^n \)

\[
\sum_{i,j=1}^{n} a_{ij}(x) D_{ij} u = 0 ,
\]

where the coefficients satisfy

\[
\lambda I \leq (a_{ij}(x)) \leq \lambda^{-1} I
\]

for some positive constant \( \lambda > 0 \), and the dimension \( n \geq 3 \). Safonov constructed homogeneous order \( \alpha \) solutions, with \( \alpha \in (0, 1) \), to (1.1) in \([9]\), where he showed the unimprovability of the estimates of the Hölder exponents for solutions to (1.1) by Krylov and himself. The homogeneity \( \alpha \) with \( \alpha < 1 \) plays an essential role in the construction. Later on, Safonov asked in \([10, \text{p. 49}]\) whether one can construct nontrivial homogeneous order 1 solutions to (1.1). Our result indicates that it is impossible to do so in \( \mathbb{R}^3 \).

**Theorem 1.1** Any homogeneous order 1 strong solution \( u \) to (1.1) in \( \mathbb{R}^3 \) with \( u \in W^{2,2}_{\text{loc}}(\mathbb{R}^3) \) must be a linear function.

On the other hand, one does have nontrivial homogeneous order 1 solutions to (1.1) in \( \mathbb{R}^4 \setminus \{0\} \). In fact, let \( (x, f(x)) \in \mathbb{R}^n \times \mathbb{R}^k \) be a nonparametric minimal surface. Then each component of \( f \) satisfies (1.1), with \( (a_{ij}(x)) \) being the inverse of the induced metric of the minimal surface in \( \mathbb{R}^{n+k} \); cf. \([6, \text{p. 3}]\). Through Hopf

fibration, Lawson and Osserman constructed a minimal cone \((x, f(x)) \in \mathbb{R}^4 \times \mathbb{R}^3\), where

\[
f(x) = \frac{\sqrt{3}}{2} \frac{1}{|x|} (x_1^2 + x_2^2 - x_3^2 - x_4^2, 2x_1x_3 + 2x_2x_4, 2x_2x_3 - 2x_1x_4)
\]

(see [6, theorem 7.1]). Now each component of \(f\) is a desired nontrivial solution to (1.1) in \(\mathbb{R}^4 \setminus \{0\}\). Actually by noticing that the graph of \(u = (x_1^2 + x_2^2 - x_3^2 - x_4^2)/|x|\) is a saddle surface, one can easily construct coefficients \(a_{ij}(x)\) in \(\mathbb{R}^4 \setminus \{0\}\) so that \(u\) satisfies (1.1) in \(\mathbb{R}^4 \setminus \{0\}\).

Theorem 1.1 gives a simple PDE proof of a well-known result obtained by many authors in the 1970s, which states that any nonparametric minimal cone of dimension 3 must be flat. Let \((x, f(x)) \in \mathbb{R}^3 \times \mathbb{R}^3\) be the minimal cone with \(f(tx) = tf(x), f \in C^\infty(\mathbb{R}^3 \setminus \{0\}), t > 0\); then each component of \(f\) satisfies (1.1). By Theorem 1.1, \(f\) must be linear, or the minimal cone is flat.

From Theorem 1.1 one also sees that any smooth homogeneous order 2 solution in \(\mathbb{R}^3 \setminus \{0\}\) to the fully nonlinear elliptic equation \(F(D^2u) = 0\) must be a quadratic polynomial. To get this conclusion, one simply applies Theorem 1.1 to the gradient \(\nabla u\) and (1.1) with \(a_{ij}(x) = \frac{\partial F}{\partial x_{ij}}(D^2u)\). In contrast, the second author constructed a nonquadratic homogeneous order 2 solution to some equation \(F(D^2u) = 0\) in \(\mathbb{R}^{12}\), which provides a counterexample to the regularity for fully nonlinear elliptic equations (see [7]).

The heuristic idea of the proof of Theorem 1.1 is the following simple geometric observation. The closed saddle surface \(\nabla u(S^2)\) must be a point. Therefore, \(u\) is linear. More precisely, one considers the surface \(\Sigma\) parametrized by the gradient \(\nabla u : S^2 \to \mathbb{R}^3\). Because of (1.1), \(\Sigma\) is a saddle surface at \(\nabla u(x)\) with \(D^2u(x) \neq 0\). However, the supporting plane with normal \(x\) touches \(\Sigma\) at \(\nabla u(x)\). Thus \(D^2u(x) \equiv 0\), and it follows that \(u\) is linear; see Section 2.

The ideas of gradient map and supporting plane are already in an early paper by Alexandrov [1]. In fact, under the assumption that the homogeneous order 1 function \(u\) is analytic in \(\mathbb{R}^3 \setminus \{0\}\) and the Hessian \(D^2u\) is either nondefinite or 0 at each point, Alexandrov showed that \(u\) must be a linear function. Roughly speaking, if \(u\) is analytic in \(\mathbb{R}^3 \setminus \{0\}\), the set \(S(u) = \{x \in S^2 : D^2u(x) = 0\}\) is either isolated or the whole \(S^2\). Alexandrov proved that the supporting plane to \(\Sigma\) is unique at \(\nabla u(x)\) with \(x\) being an isolated point of \(S(u)\). Since the surface \(\Sigma\) has supporting planes with normal along all the directions in \(\mathbb{R}^3\), Alexandrov excluded the case of \(S(u)\) being isolated and proved the result. It is interesting to note that the concept of gradient maps and supporting planes was further employed in the later development of the Alexandrov-Bakel’man-Pucci maximum principle.

When the coefficients \(a_{ij}\) are \(C^\alpha\) and the solution \(u\) is \(C^{2,\alpha}\) away from the origin, we can show that the set \(S(u)\) is either isolated or the whole \(S^2\). Coupled with Alexandrov’s argument, one sees that Theorem 1.1 holds in the \(C^{2,\alpha}\) setting.
After the work in the $C^{2,\alpha}$ case was done, we found that Pogorelov [8] generalized Alexandrov’s result above to $C^2$ functions. The argument in [8] is more involved. Since our approach in the $C^{2,\alpha}$ case is interesting in its own right as well as short, we include it here in Section 3.

2 Proof of the Main Theorem

Let $h(x_1, x_2) = u(x_1, x_2, 1)$. Then the homogeneous order 1 function $u(x_1, x_2, x_3) = x_3 h(x_1/x_3, x_2/x_3)$, the gradient $\nabla u$, and the Hessian $D^2u$ have the following representation:

\begin{equation}
\nabla u(x_1, x_2, 1) = (h_1, h_2, h - x_1 h_1 - x_2 h_2),
\end{equation}

\begin{equation}
D^2u(x_1, x_2, 1) = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-x_1 & -x_2 & 1
\end{bmatrix}
\begin{bmatrix}
h_{11} & h_{12} & 0 \\
h_{21} & h_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0 & -x_1 \\
0 & 1 & -x_2 \\
0 & 0 & 1
\end{bmatrix}.
\end{equation}

From our assumption that the homogeneous order 1 solution $u$ is in $W^{2,2}_{\text{loc}}(\mathbb{R}^2)$, it follows that $h \in W^{2,2}_{\text{loc}}(\mathbb{R}^2)$ is a strong solution to

\begin{equation}
\sum_{i,j=1}^{2} A_{ij}(x_1, x_2) D_{ij} h = 0,
\end{equation}

where the coefficients $A_{ij}$ are in terms of $a_{ij}(x_1, x_2, 1)$, $x_1$, and $x_2$ and satisfy the ellipticity condition with some $\lambda(x_1, x_2)$.

**Lemma 2.1** For any $v \in \mathbb{S}^2$, the supporting plane with normal $v$ must touch the surface $\Sigma$ parametrized by $\nabla u : \mathbb{S}^2 \to \mathbb{R}^3$ at $\nabla u(v)$ or $\nabla u(-v)$.

**Proof:** First we notice that $\nabla u \in C^\alpha(\mathbb{S}^2)$. In fact, each component of $(h_1, h_2)$, say $h_1$, is a $W^{1,2}_{\text{loc}}(\mathbb{R}^2)$ weak solution to a divergence equation

\[\sum_{i,j=1}^{2} D_i (B_{ij} D_j h) = 0,
\]

where $B_{11} = A_{11}/A_{22}$, $B_{12} = (A_{12} + A_{21})/A_{22}$, $B_{21} = 0$, and $B_{22} = 1$. It follows that $h \in C^{1,\alpha}(\mathbb{S}^2)$ (cf. [5, pp. 84–285 or theorem 12.4]). By (2.1) we see $\nabla u \in C^\alpha(\mathbb{S}^2)$ with $\alpha$ depending only on the original ellipticity $\lambda$. Therefore, for any $v \in \mathbb{S}^2$, the supporting plane $P_v$ with normal $v$ must touch the surface $\Sigma$ somewhere ($\Sigma$ is on the opposite side of $v$). Without loss of generality, we assume $v = (1, 0, 0)$.

**Claim.** $P_{(1,0,0)}$ touches $\Sigma$ at $\nabla u(x_1, x_2, 0)$ with $(x_1, x_2, 0) \in \mathbb{S}^2$.

Suppose $P_{(1,0,0)}$ touches $\Sigma$ at $\nabla u(x_1, x_2, x_3)$ with $x_3 \neq 0$, say $x_3 > 0$. Then $u_1(x_1, x_2, 1) = u_1(x_1/x_3, x_2/x_3, 1) = h_1(x_1/x_3, x_2/x_3)$ would achieve its maximum at $(x_1/x_3, x_2/x_3)$. Because of (2.3), $h_1$ satisfies the strong maximum principle (cf. [5, theorem 8.19]). We then have a contradiction unless $h_1 \equiv \text{const}$. In the
latter case, \( u_1(x_1, x_2, x_3) \equiv \text{const} \) for \( x_3 > 0 \), and our claim holds. Similarly, by applying the above argument to \( u(x_1, 1, x_3) \), we see that \( P_{(1,0,0)} \) also touches \( \Sigma \) at \( \nabla u(x_1, 0, x_3) \) with \( (x_1, 0, x_3) \in S^2 \). Therefore, \( P_{(1,0,0)} \) must touch \( \Sigma \) at \( \nabla u(1, 0, 0) \) or \( \nabla u(-1, 0, 0) \).

**Proof of Theorem 1.1**: By our assumption on \( u, u \in W^{2,2}(S^2) \). If \( D^2u = 0 \) almost everywhere on \( S^2 \), then \( u \) is already linear. Otherwise, we pick a Lebesgue point \( x^* \in S^2 \) for \( D^2u \) with \( D^2u(x^*) \neq 0 \), say \( x^* = (0, 0, 1) \). We may also assume \( x^* \) is a Lebesgue point for \( \nabla u \) and \( (u_{ij}(x)) \). By Lemma 2.1, the supporting planes \( \Pi_{(0,0,1)} \) and \( \Pi_{(0,0,1)} \) touch \( \Sigma \) at \( \nabla u(0, 0, 1) \) or \( \nabla u(0, 0, -1) \). If both planes touch \( \Sigma \) at the same point, then we see that \( u_3 \equiv c \). Consequently, the homogeneous order 1 function \( v(x_1, x_2) = u - cx_3 \) satisfies (1.1), and it follows that \( v \) is linear, or \( u \) is also linear. So we are left with the case that \( P_{(0,0,1)} \) and \( P_{(0,0,1)} \) touch \( \Sigma \) at different points. We may assume \( P_{(0,0,1)} \) touches \( \Sigma \) at \( \nabla u(0, 0, 1) \). It means that

\[
(2.4) \quad u_3(x_1, x_2, 1) = h - x_1h_1 - x_2h_2 \leq h(0) \quad \text{near} \quad (0, 0).
\]

By (2.1) and (2.2), \((0, 0)\) is a Lebesgue point for \( D^2h \) and \( \nabla h \). It follows that (cf. [4, app. C])

\[
h(x_1, x_2) = h(0) + h_1(0)x_1 + h_2(0)x_2 + \frac{1}{2} \sum_{i,j=1}^2 h_{ij}(0)x_ix_j + o(|x|^2).
\]

Because of (2.3) and \( D^2h(0, 0) \neq 0 \), we may assume \( h_{11}(0) = a, h_{12}(0) = 0, \) and \( h_{22}(0) = -b \) with \( a, b > 0 \). Simple integration of (2.4) yields

\[
h(0)t + \frac{1}{6}bt^3 + o(t^3) = \int_0^t u_3(0, x_2, 1)dx_2 \leq h(0)t
\]

for \( t > 0 \). The impossibility of the above inequality completes the proof. \( \Box \)

### 3 Another Proof in the \( C^{2,\alpha} \) Case

In this section, we present yet another proof of Theorem 1.1 in the \( C^{2,\alpha} \) case. Suppose \( u : \mathbb{R}^3 \setminus \{0\} \to \mathbb{R} \) is a \( C^{2,\alpha} \) homogeneous order 1 function \( u = rg(\theta_1, \theta_2) \), where \((r, \theta_1, \theta_2)\) are the spherical coordinates with \( x_1 = r \cos \theta_2 \cos \theta_1, \ x_2 = r \cos \theta_2 \sin \theta_1, \ x_3 = r \sin \theta_2 \). Then the Hessian \( D^2u = \left( \frac{\partial^2 u}{\partial x_i \partial x_j} \right) \) has the following representation:

\[
D^2u(x) = \frac{1}{r} R(\theta_1, \theta_2) H R'(\theta_1, \theta_2),
\]

where

\[
H = \begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{1}{\cos^2 \theta_2} \frac{\partial^2 g}{\partial \theta_1^2} - \tan \theta_2 \frac{\partial g}{\partial \theta_2} + g & \frac{\partial^2 g}{\partial \theta_1 \partial \theta_2} + \tan \theta_2 \frac{\partial g}{\partial \theta_1} \\
0 & \frac{\partial^2 g}{\partial \theta_1 \partial \theta_2} + \tan \theta_2 \frac{\partial g}{\partial \theta_1} & \frac{\partial^2 g}{\partial \theta_2^2} + g
\end{bmatrix}
\]
and \( R(\theta_1, \theta_2) \) is the rotation from \((\partial x_1, \partial x_2, \partial x_3)\) to \((\partial r, \frac{1}{r}\partial \theta_1, \frac{1}{r}\partial \theta_2)\). We see that \( D^2u = 0 \) if and only if \( H = 0 \). We define the singular set \( S(u) \) as

\[
S(u) = \{ x \in \mathbb{S}^2 : D^2u(x) = 0 \}.
\]

**Lemma 3.1** Let \( u \) be as in Theorem 1.1. Then one of the following holds:

(i) \( S(u) \) is empty,
(ii) \( S(u) \) consists of finitely many points, or
(iii) \( S(u) = \mathbb{S}^2 \).

**Proof:** Suppose \( S(u) \) is not empty. For any \( p \in S(u) \), we prove that either \( p \) is isolated or \( S(u) \) contains a neighborhood of \( p \) on \( \mathbb{S}^2 \). Without loss of generality, we assume \( p = (1, 0, 0) \) and \( a_{ij}(p) = \delta_{ij} \). Note that \( S(u) = S(u+ax_1+bx_2+cx_3) \); we may also assume \( \nabla u(p) = 0 \). Then \( u(p) = p \cdot \nabla u(p) = 0 \). Correspondingly, we have

\[
\left( \frac{\partial^2 g}{\partial \theta_1 \partial \theta_2} \right)(0, 0) = 0, \quad \left( \frac{\partial g}{\partial \theta_1}, \frac{\partial g}{\partial \theta_2} \right)(0, 0) = 0, \quad \text{and} \quad g(0, 0) = 0.
\]

Also \( g \) satisfies

\[
\sum_{i,j=1}^2 A_{ij}(\theta_1, \theta_2) \frac{\partial^2 g}{\partial \theta_i \partial \theta_j} + \sum_{i=1}^2 B_i(\theta_1, \theta_2) \frac{\partial g}{\partial \theta_i} + C(\theta_1, \theta_2) g = 0,
\]

where the \( C^\alpha \) coefficients \( (A_{ij}) \) satisfy the ellipticity condition with the same constant \( \lambda \), \( A_{ij}(0, 0) = \delta_{ij} \), and \( B_i \) and \( C \) are \( C^\alpha \) functions near \((0, 0)\). Suppose \( g \) vanishes up to infinity order at \((0, 0)\); then by the Carleman unique continuation (cf. [3, p. 124]), \( g \equiv 0 \) in a neighborhood of \((0, 0)\). It follows that \( D^2u = 0 \) on \( \mathbb{S}^2 \), since \( u \) is \( C^2 \). Suppose \( g \) vanishes up to order \( k - 1 \) at \((0, 0)\). By our assumption \( k \geq 3 \). We apply the result of [2, theorem 1] to obtain

\[
g = P(\theta_1, \theta_2) + R(\theta_1, \theta_2),
\]

where the homogeneous order \( k \) polynomial \( P \) satisfies \( \sum_{i,j=1}^2 A_{ij}(0, 0) \frac{\partial^2 P}{\partial \theta_i \partial \theta_j} = 0 \), or \( \Delta P = 0 \), and the remainder satisfies

\[
R \sim O(\vert \theta \vert^{k+\alpha}), \quad \nabla R \sim O(\vert \theta \vert^{k-1+\alpha}), \quad D^2 R \sim O(\vert \theta \vert^{k-2+\alpha}),
\]

with \( \alpha \in (0, 1) \). It is a simple fact that \( \{ (\theta_1, \theta_2) : D^2 P = 0 \} = \{(0, 0)\} \), and consequently \( \{ (\theta_1, \theta_2) : D^2 H = 0 \} = \{(0, 0)\} \). Hence \( p \) is an isolated zero point of \( D^2 u \). Therefore \( S(u) \) consists of finitely many points in this case. \( \square \)

We consider the surface \( \Sigma \) parametrized by \( \nabla u : \mathbb{S}^2 \to \mathbb{R}^3 \). From (3.1), it follows that the Hessian always has one zero eigenvalue. Let \( \lambda_1(x) \) and \( \lambda_2(x) \) be the other two eigenvalues of \( D^2u(x) \). Because of equation (1.1), \( \lambda_1(x)\lambda_2(x) < 0 \) for all \( x \in \mathbb{S}^2 \setminus S(u) \).

**Lemma 3.2** For any \( x \in \mathbb{S}^2 \setminus S(u) \), the surface \( \Sigma \) is \( C^{2,\alpha} \) at \( \nabla u(x) \) with a normal vector given by \( x \) and the two principal curvatures given by \(-1/\lambda_1(x)\) and \(-1/\lambda_2(x)\).
PROOF: We may assume \( x = p = (0, 0, 1) \in \mathbb{S}^2 \setminus S(u) \). Then locally at \( \nabla u(p) \), \( \Sigma \) can be represented by

\[
F(x_1, x_2) = \nabla u \left( x_1, x_2, \sqrt{1 - x_1^2 - x_2^2} \right).
\]

By differentiating the identity \( x \cdot \nabla u(x) = u(x) \) twice with respect to \( x_i, x_j \) for \( i, j = 1, 2 \), we obtain

\[
\begin{align*}
\frac{\partial}{\partial x_i} u_{3j}(p) = 0, & \quad \frac{\partial}{\partial x_i} u_{3ij}(p) = -u_{ij}(p) .
\end{align*}
\]

By differentiating \( (3.2) \) and making use of \( (3.3) \), we have

\[
F_i(0) = (u_{11}(p), u_{12}(p), 0) \quad \text{and} \quad F_{ij}^{(3)}(0) = -u_{ij}(p),
\]

where \( F_{ij}^{(3)} \) denotes the third component of the vector \( F_{ij} \). Then we get

\[
F_1(0) \times F_2(0) = (0, 0, (u_{11}u_{22} - u_{12}^2)(p)) .
\]

It is easy to see that \( (u_{11}u_{22} - u_{12}^2)(p) = \lambda_1\lambda_2(p) \neq 0 \). Hence \( (0, 0, 1) \) is a normal vector for \( \Sigma \) at \( \nabla u(p) \). We see that \( \nabla u = G^{-1} \) near \( (0, 0, 1) \), where \( G^{-1} \) is the inverse of the Gauss map of \( \Sigma \). Thus \( \Sigma \) is \( C^{2,\alpha} \) nearby. We also get the first and second fundamental forms of \( \Sigma \) at \( \nabla u(p) \) as follows:

\[
I = (u_{11}^2 + u_{22}^2)dx_1^2 + 2u_{12}(u_{11} + u_{22})dx_1 \, dx_2 + (u_{21}^2 + u_{22}^2)dx_2^2 ,
\]

\[
II = -u_{11} \, dx_1^2 - 2u_{12} \, dx_1 \, dx_2 - u_{22} \, dx_2^2 ,
\]

where all \( u_{ij} \) are evaluated at \( p \). Therefore the two principal curvatures are \(-1/\lambda_1 \) and \(-1/\lambda_2 \). \( \square \)

Remark 3.3. In computing the principal curvatures of \( \Sigma \) in terms of the eigenvalues of \( D^2u \), we differentiate \( u \) three times. By approximation, the conclusion still holds for \( u \in C^{2,\alpha} \).

Now we are ready for another proof of Theorem 1.1 in the \( C^{2,\alpha} \) case.

PROOF: We prove that \( S(u) = \mathbb{S}^2 \) by excluding cases (i) and (ii) in Lemma 3.1. We assume \( S(u) \) consists of at most finitely many points. First, the gradient surface \( \Sigma \) has supporting planes with normals along all the directions in \( \mathbb{R}^3 \). However, the saddle points in \( \{ \nabla u(x) : D^2u(x) \neq 0 \} \) cannot support any supporting planes. Hence there are at most finitely many points on \( \Sigma \) with the supporting planes.

Claim (Alexandrov). For any \( x_0 \in S(u) \), the supporting plane to \( \Sigma \) at \( \nabla u(x_0) \) can only have normal direction \( x_0 \).

Therefore there are at most finitely many supporting planes to \( \Sigma \). This is a contradiction. Now we prove the claim. Suppose there is another supporting plane \( P_1 \) to \( \Sigma \) at \( \nabla u(x_0) \) with normal direction \( x_1 \neq x_0 \). Take a small neighborhood \( U \) of \( x_0 \) on \( \mathbb{S}^2 \) such that \( D^2u(x) \neq 0 \) for any \( x \neq x_0 \in U \) and \( U \cap S^* = \phi \), with \( S^* \) being a great circle through \( x_1 \) and \( -x_1 \). This can be done since \( x_0 \) is an isolated point in \( S(u) \). Now lift \( P_1 \) along the \( x_1 \)-direction to \( P \) so that \( P \cap \nabla u(U) = C \) is a smooth
close curve on $P$. We can take a point on $C$, say $x^* \in U$ and $x^* \neq x_0$, such that the normal to the plane curve $C$ at $\nabla u(x^*)$ is along the intersection $P$ and another plane through $S^*$. Then we see that the normal of the surface $\Sigma$ at the regular point $\nabla u(x^*)$ must be on $S^*$. On the other hand, the normal at $\nabla u(x^*)$ is $x^* \in U$. This contradiction completes the proof of the claim. By excluding cases (i) and (ii) in Lemma 3.1, we are left with case (iii), $S(u) = S^2$. That is $D^2u \equiv 0$, and hence $u$ is linear. This finishes the proof of Theorem 1.1 in the $C^{2,\alpha}$ case.  

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