Weekly Homework 1

Due: Friday, Jan 15 2016

January 8, 2016

Problem 1 (Solutions To Differential Equations). For each of the following, show whether or not the specified function is a solution to the corresponding differential equation.

(a) $y'''' + y''' + y' - y = 0$, $y(x) = \cos(x)$

(b) $\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + 6u \frac{\partial u}{\partial x} = 0$, $u(x, t) = \frac{1}{2} \csc h^2 \left[ \frac{\sqrt{c}}{2} (x - ct - a) \right]$

(c) $y'' - y = 0$, $y(x) = \sinh(x)$

Problem 2 (Solving differential equations). For each of the following differential equations, do the following

(i) Identify the type of differential equation
(ii) Find the “general solution”

(a) $y' = 2y + 3$

(b) $y' = \frac{x^2 - y^2}{x + y}$

(c) $\sin(u) \frac{du}{dt} = \cos(u)/(1 + t^2)$

(d) $\frac{dy}{dt} = \frac{t^2 - y^2}{ty}$

(e) $(3x - 4y)dy = (2x + 7y)dx$

(f) $\frac{du}{dt} + y/t = 6 \cos(4t)$

(g) $y' + y = \cos(t)$

(h) $y' = 1 - y^3$

Problem 3 (Waaaaait a minute!). Explain what is wrong with the following argument:
Consider the differential equation

\[ y' = 1 - 2y \]

Integrating both sides, we get the equation

\[ y = y - y^2 + C. \]

Simplifying this, we get the solution \( y^2 = C \) meaning that

\[ y = \pm \sqrt{C}. \]

Problem 4 (Slope fields). For each of the following initial value problems

(i) Plot the slope field

(ii) Based on the plot of the slope field, predict the behavior of a solution to the IVP at large values of \( t \)

(iii) Explicitly solve the IVP

(iv) Based on the explicit solution of the IVP, determine the behavior at large values of \( t \)

(a) \( y' = y(1 - y^2), \ y(0) = 1 \)

(b) \( y' = y(1 - y^2), \ y(0) = 1/2 \)

(c) \( y' = y(1 - y^2), \ y(0) = 3/2 \)

Problem 5 (Second order equations). Consider the second order differential equation

\[ y'' - y = 0 \]

(a) Show that the change of variables \( z = y' + y \) in the above second-order equation transforms it into the first order equation

\[ z' - z = 0 \]

(b) Find the general solution of the first-order equation of (a)

(c) By substituting the value of \( z \) back into the equation \( z = y' + y \), find the value of \( y \). Your final answer for \( y \) should involve two arbitrary constants.

Problem 6 (Solving Initial Value Problems). Find a solution to each of the following initial value problems

(a) \( y' = x \cos(y), \ y(0) = 1 \)

(b) \( y' = e^x + y, \ y(1) = 2 \)
(c) \( \frac{dy}{dt} + 2y = te^{-2t}, \quad y(1) = 0 \)

(d) \( xy' + 2y = \sin(x), \quad y(\pi/2) = 1 \)

**Problem 7 (An almost homogeneous equation).** Consider the differential equation

\[ y' = x \cos(y/x) + y/x \]

(a) Explain why this is not a homogeneous differential equation

(b) Find the general solution of the differential equation.