

Math 521-721 Qual Sequence Syllabus

Topics Covered

- Group Theory
 - Basic Group Theory: Definition of group, Subgroups, Cayley's Theorem, Lagrange's Theorem, Key examples of groups.
 - Quotient Groups: Cosets, Normal Subgroups, Isomorphism Theorems
 - Group Actions: Action of a group on itself, Conjugacy classes, the class equation
 - Sylow Theorems: Proofs using group actions, applications to groups of small order
- Ring Theory
 - Basic Ring theory: units, zero divisors, types of rings (integral domains, fields, etc.) and key examples (polynomial rings, matrix rings)
 - Quotient rings and ideals: Types of ideals (prime ideals, maximal ideals), isomorphism theorems, adjoining elements
 - Factorization in rings. PIDs and UFDs
 - Integral domains, field of fractions, localization
- Field Theory
 - Algebraic and transcendental field elements, adjoining roots of equations
 - Degree of a field extension, the tower law, applications to ruler and compass constructions
 - Galois groups, the fundamental theorem of Galois theory
 - Simple groups, and solvability by radicals
 - Finite fields, transcendental extensions, the transcendence degree
- Module Theory
 - Module basics, free modules and modules given by generators and relations, direct sum, tensor product and Hom.
 - Noetherian rings and Modules, Hilbert basis theorem
 - Structure theory of modules over a PID, Smith normal form, applications of structure theory to abelian groups and linear algebra
 - Tensor, exterior, and symmetric algebra
 - Complexes and cocomplex of modules, exact sequences, diagram chases, homology and cohomology
 - Primary decomposition in Noetherian rings and modules
- Representation Theory of Finite Groups
 - Definitions of representations, modules over the group ring
 - Irreducible representations and Schur's Lemma, Wedderburn's Theorem
 - Character theory and the orthogonality relations
 - Introduction to representation theory of the symmetric group

References

Dummit and Foote, *Abstract Algebra*, 3rd edition. Chapters 1-5, 7-8, 10-14, 15.1-15.2, 17.1, 18