Math 721 – Homework 5
Due Friday, February 21 at 5pm

Good practice problems (do not turn in solutions):
DF 12.1 Exercises 1, 2, 3, 4, 5, 8, 9, 20, 21
DF 12.2 Exercises 1, 2, 5, 6, 21

Problem 1 (DF 12.1 Exercise 6). Show that if R is an integral domain and M is any non-principal ideal of R, then as an R-module, M is torsion-free of rank 1, but is not a free R-module.

Problem 2 (DF 12.2 Exercises 3, 4). Let F be a field.
(a) Prove that two 2×2 matrices over F which are not scalar multiple of the identity matrix are similar if and only if they have the same characteristic polynomial.
(b) Prove that two 3×3 matrices over F are similar if and only if they have the same characteristic and minimal polynomials.
(c) Give an explicit counterexample to the statement of (b) for 4×4 matrices.