Math 514 - Homework 8

Due on Thursday, December 5

You are welcome to talk with other students in the class about problems but should write up solutions on your own. Solutions can be handwritten or typed but need to be legible and submitted via Gradescope by the end of the day on Thursday. You should justify all your answers in order to receive full credit.

Problem 1. Exercise 10.4 from Chapter 10 Schrijver's notes.

Problem 2. Exercise 10.17 from Chapter 10 Schrijver's notes.

Hint: The linear matroid defined by the n columns of a $d \times n$ matrix M is the same as that of UM for any invertible $d \times d$ matrix U. Also, if $M = \begin{pmatrix} I_d & A \end{pmatrix}$ for some $d \times (n-d)$ matrix A, then there is a relationship between the $d \times d$ minors of M and the $(n-d) \times (n-d)$ minors of the $(n-d) \times n$ matrix $\begin{pmatrix} A^T & I_{n-d} \end{pmatrix}$.

Problem 3. Exercise 10.18 from Chapter 10 Schrijver's notes.

Problem 4. Let $G = (V, E)$ be a connected graph. In Problem 4 of Homework 2, you considered the polytope

$$P_G = \left\{ x \in \mathbb{R}^E_{\geq 0} : \sum_{e \in E} x_e = |V| - 1 \text{ and } \sum_{e \in E(S)} x_e \leq |S| - 1 \text{ for all } S \subset V \text{ with } |S| \geq 2 \right\}$$

where $E(S)$ denotes the set of edges both of whose vertices belong to S. For a subset $T \subseteq E$, let $\mathbf{1}_T$ denote the vector in $\mathbb{R}^E$ with $(\mathbf{1}_T)_e = 1$ for $e \in T$ and $(\mathbf{1}_T)_e = 0$ for $e \notin T$.

Use Corollary 10.14a to show that P_G equals the matroid polytope of the graphic matroid of G, i.e. $\text{conv}\{\mathbf{1}_T : T \text{ is a spanning tree of } G\}$.