Math 514 - Homework 8

Due on Thursday, November 21

You are welcome to talk with other students in the class about problems but should write up solutions on your own. Solutions can be handwritten or typed but need to be legible and submitted via Gradescope by the end of the day on Thursday. You should justify all your answers in order to receive full credit.

Problem 1. Exercise 8.12 from Chapter 8 Schrijver's notes.
Remark: you need to show this for every non-bipartite graph, not just a one example.

Problem 2. Exercise 8.16 from Chapter 8 Schrijver's notes.

Problem 3. Exercise 8.19 from Chapter 8 Schrijver's notes.

Problem 4. Let $D = (V, E)$ be a directed graph with $s, t \in V$ and let $c : E \to \mathbb{R}_{\geq 0}$ and $k : E \to \mathbb{R}_{\geq 0}$. As described in Section 4.6, the cost of an $s - t$ flow $f : E \to \mathbb{R}_{\geq 0}$ is

$$\text{cost}(f) = \sum_{e \in E} k(e) f(e).$$

The min-cost max-flow problem is to find a minimum cost flow among the set of $s - t$ flows with maximum value.

(i) Describe how to solve the min-cost max-flow problem using linear programming (by solving one or more LPs).

(ii) For the example in Exercise 4.16(i) from Chapter 4 Schrijver's notes, find all vertices of the polytope of maximum-value flows and find a maximum-value flow of minimum cost.
Hint: you might want to reduce the number of parameters before computing vertices.

(iii) Use total unimodularity to show that when $c : E \to \mathbb{Z}_{\geq 0}$, there is an integer-valued flow $f : E \to \mathbb{Z}_{\geq 0}$ that achieves the minimum cost among all $s - t$ flows of maximum value.