Math 514 - Homework 7

Due on Thursday, November 14

You are welcome to talk with other students in the class about problems but should write up solutions on your own. Solutions can be handwritten or typed but need to be legible and submitted via Gradescope by the end of the day on Thursday. You should justify all your answers in order to receive full credit.

Problem 1. Exercise 8.4 from Chapter 4 Schrijver's notes.

Problem 2. (not graded this week, moved to HW 8)
Exercise 8.19 from Chapter 8 Schrijver's notes.

Problem 3. Let $A \in \mathbb{Z}^{m\times n}$ with rank n. Show that A is unimodular if and only if for every $b \in \mathbb{Z}^m$, every vertex of $\{x \in \mathbb{R}^n : Ax \le b\}$ is integer.
Remark: Here we say A is unimodular if every $n \times n$ submatrix has determinant 0, ± 1. This is the transpose of the definition in the notes (switching m and n).

Problem 4. Give an example of each of the following.

(i) A unimodular matrix $A \in \mathbb{R}^{m\times n}$ and vector $b \in \mathbb{Z}^m$ for which $\{x \in \mathbb{R}^n : Ax \le b\}$ has a non-integer vertex.

(ii) A matrix $A \in \mathbb{Z}^{m\times n}$ and vector $b \in \mathbb{Z}^m$ for which every vertex of the polyhedron $\{x \in \mathbb{R}^n : Ax \le b\}$ is integer but the matrix A is not totally unimodular.