

## Math 514 - Homework 4

Due on Thursday, October 24

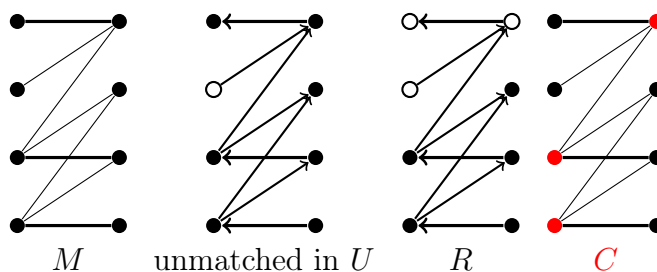
You are welcome to talk with other students in the class about problems but should write up solutions on your own. Solutions can be handwritten or typed but need to be legible and submitted via Gradescope by the end of the day on Thursday. You should justify all your answers in order to receive full credit.

**Problem 1.** Exercise 3.11 from Chapter 3 of Schrijver's notes.

*Remark:* You will later *use* this to show the inequality description of the matching polytope. So you should not use this theorem in your proof!

**Problem 2.** Exercise 3.17 from Chapter 3 of Schrijver's notes.

**Problem 3.** (Exercise 3.19) Let  $G = (U \uplus W, E)$  be a bipartite graph and  $M \subseteq E$  a matching of maximum size. Let  $D = (U \uplus W, A)$  be the directed graph used when looking for an  $M$ -augmenting path and let  $R$  denote the set of vertices in  $D$  reachable from an unmatched vertex in  $U$ . Show that  $C = (U \setminus R) \cup (W \cap R)$  is a vertex cover of  $G$  with  $|M| = |C|$ .



**Problem 4.** Exercise 3.25 from Chapter 3 of Schrijver's notes.