Math 514 - Homework 2

Due on Thursday, October 10

You are welcome to talk with other students in the class about problems but should write up solutions on your own. Solutions can be handwritten or typed but need to be legible and submitted via Gradescope by the end of the day on Thursday. You should justify all your answers in order to receive full credit.

Problem 1. Exercise 2.2 from Chapter 2 of Schrijver's notes.

Problem 2. Exercise 2.4 from Chapter 2 of Schrijver's notes.

Problem 3. Exercise 2.6(ii) from Chapter 2 of Schrijver's notes.

Problem 4. Let $G = (V, E)$ be a connected graph and consider the polytope

$$P_G = \left\{ x \in \mathbb{R}^E_{\geq 0} : \sum_{e \in E} x_e = |V| - 1 \text{ and } \sum_{e \in E(S)} x_e \leq |S| - 1 \text{ for all } S \subset V \text{ with } |S| \geq 2 \right\}$$

where $E(S)$ denotes the set of edges both of whose vertices belong to S. For a subset $T \subseteq E$, let $\mathbf{1}_T$ denote the vector in $\mathbb{R}^E$ with $(\mathbf{1}_T)_e = 1$ for $e \in T$ and $(\mathbf{1}_T)_e = 0$ for $e \notin T$.

(a) Show that for any spanning tree T of G, $\mathbf{1}_T$ belongs to P_G.

(b) Show that any integer point in P_G has the form $\mathbf{1}_T$ for some spanning tree T of G.

Remark: In fact, the vertices of P_G are precisely $\{\mathbf{1}_T : T \text{ is a spanning tree of } G\}$, but you do not have to prove this.