

Math 407 – Homework 8

Due on Thursday, December 7

You are welcome to talk with other students in the class about problems but should write up solutions on your own. Solutions can be handwritten or typed but need to be legible and submitted via Gradescope by the end of the day on Thursday. You should justify all your answers in order to receive full credit.

Problem 1. This problem refers to the linear program $\max\{\mathbf{c}^T \mathbf{x} : A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0\}$ where

$$A = \begin{pmatrix} 1 & -1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \quad \text{and} \quad \mathbf{c}^T = (4 \ 1 \ 2 \ 0 \ 0).$$

The optimal solution is $\mathbf{x}^* = (1, 0, 1, 0, 0)$ with basis $B = \{1, 3\}$.

- For what values of $t \in \mathbb{R}$, does $\{1, 3\}$ remain a feasible basis corresponding to an optimal solution after changing \mathbf{c} to $\tilde{\mathbf{c}} = \mathbf{c} + (0, 0, t, 0, 0)^T$?
- For t in this range, what is the optimal value?
- For what values of $t \in \mathbb{R}$, does $\{1, 3\}$ remain a feasible basis corresponding to an optimal solution after changing \mathbf{b} to $\tilde{\mathbf{b}} = \mathbf{b} + (t, 0)^T$?
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Problem 2 (Chvátal, Problem 10.2 (i)(ii)(iii), see also the class notes from Oct. 20).

A furniture-manufacturing company makes bookcases, desks, chairs, and bedframes.

- A bookcase requires three hours of work, one unit of metal, and four units of wood, and brings in a net profit of \$19.
- A desk requires two hours of work, one unit of metal, and three units of wood, and brings in a net profit of \$13.
- A chair requires one hour of work, one unit of metal, and three units of wood, and brings in a net profit of \$12.
- A bedframe requires two hours of work, one unit of metal, and four units of wood, and brings in a net profit of \$17.
- 225 hours of labor, 117 units of metal, and 420 units of wood are available per day.

Under these conditions, the company found that using their resources to make 39 bookcases, 48 chairs, and 30 bedframes each day maximized their profit. Solve each of the following variations. (That is, find the number of each product company should make per day to maximize profit.)

- The net profit brought in by each desk increases from \$13 to \$15.
- The availability of metal increased from 117 to 125 units per day
- The company may also produce coffee tables, each of which require three hours of work, one unit of metal, two units of wood, and brings in a net profit of \$14.

Problem 3. Please take a few minutes to fill out course evaluations! Your (constructive) feedback is appreciated. You do not need to turn in anything for this part of the homework.