Although it remains unknown how Gauss found the invariant measure for the regular continued fractions interval map, one can determine this measure by way of an appropriate cross-section for the geodesic flow on the unit tangent bundle of the modular surface. With P. Arnoux, we show that a reasonably large class of uniformly expansive piecewise maps have invariant measures induced as the marginal measure of a higher dimensional system, which is then the natural extension. The higher dimensional system can be described in terms of iterated function systems and can often be explicitly determined by plotting the orbit of an arbitrary point. In the case of piecewise Möbius interval maps, entropy conditions then determine if the natural extension can be given as a cross-section for geodesic flow.

In the talk, we’ll first concentrate on the motivation, explicitly giving the Gauss measure and the corresponding cross-section. Other continued fraction interval maps will be used to illustrate the entropy conditions, and we will then turn to the existence proof.