In Federer’s standard text of Geometric Measure Theory he states the development of GMT was largely motivated by the conjecture that sets with positive finite density are rectifiable. This conjecture is a special case of Preiss’s spectacular theorem that Radon measures whose density is positive and finite are supported on a rectifiable set. A precursor to Preiss’s Theorem is the following beautiful theorem of Marstrand

**Theorem 1 (Marstrand 1964)** Let $\mu$ be a Radon measure on $\mathbb{R}^n$ with the property that

$$0 < \lim_{r \to 0} \frac{\mu(B_r(x))}{r^s} < \infty \text{ for } \mu \text{ a.e.}$$

then $s$ is an integer.

Marstrand’s methods and later Preiss’s methods all rely heavily on the structure of the Euclidean norm. For that reason a key model problem was to prove rectifiability results for measures whose density exists with respect to the sup norm, since for the sup norm all known methods from Euclidean space break down. Another motivation was that since a polytope can be realized as a slice through a high enough dimensional cube, proving general results for the sup norm is equivalent to proving them for polytope density.

We will survey the area, briefly describing the key techniques in Euclidean space and the more recent advances starting with results for the sup norm, then polytope density sketching the main ideas and open problems.