Homework to be done by November 18th

Section 7.6: Problems 13, 19, 29.
Section 7.7: Problems 21, 25, 27, 37, 41, 43, 49, 55, 60, 67.
Section 7.8: Problems 5, 9, 17, 29, 37, 43, 48.
Section 7.9: Problems 14, 19, 20, 21, 37.

Problems to be handed in on November 18th

Problem 1. Consider the differential equation

\[ y'' + \lambda y = 0. \]

1.1 Check that \( y = ax \) satisfies (*) when \( \lambda = 0 \). Here \( a \in \mathbb{R} \).

1.2 Check that \( y = a \cos(\sqrt{\lambda} x) + b \sin(\sqrt{\lambda} x) \) satisfies (*) when \( \lambda > 0 \). Here \( a, b \in \mathbb{R} \).

1.3 Check that \( y = a \cosh(\sqrt{-\lambda} x) + b \sinh(\sqrt{-\lambda} x) \) satisfies (*) when \( \lambda < 0 \). Here \( a, b \in \mathbb{R} \).

Problem 2. Let

\[ f(x) = \tan^{-1} \left( \frac{a + x}{1 - ax} \right), \quad x \neq \frac{1}{a}. \]

2.1 Show that

\[ f'(x) = \frac{1}{1 + x^2}, \quad x \neq \frac{1}{a}. \]

2.2 Show that there is no constant \( C \) such that

\[ f(x) = \tan^{-1} x + C \quad \text{for all} \quad x \neq \frac{1}{a}. \]

2.3 Find constants \( C_1 \) and \( C_2 \) such that

\[ f(x) = \tan^{-1} x + C_1 \quad \text{for} \quad x > \frac{1}{a}, \]

and

\[ f(x) = \tan^{-1} x + C_2 \quad \text{for} \quad x < \frac{1}{a}. \]