Homework to be done by October 14th

Section 3.5: Problems 19, 23, 35, 39, 41, 44, 49, 51, 63, 64, 68, 72.
Section 3.6: Problems 5, 7, 9, 27, 29.
Section 3.7: Problems 4, 6, 11, 19, 45, 59.
Section 3.9: Problems 3, 5, 18, 22.

There are no problems from Section 3.4, which introduces various applications of the derivative. Very likely, you have seen most of them before, nevertheless you should browse through it. Section 3.8 discusses “related rates” problems, which are “practical” applications of the chain rules. We do not discuss it in any detail.

Problems to be handed in on October 14th

1. Suppose that $f$ and $g$ are twice differentiable. Compute

$$
\frac{d}{dx} \left[ f \left( \frac{3}{x} \right) \frac{d}{dx} g (x^4 - 5x) \right]
$$

in terms of the derivatives of $f$ and $g$.

2. In the park there are 2 intersecting paths, one going north-south and the other going east-west. At a certain time, Bill is 40 feet north of the intersection and is walking south at 4ft/sec, and Hillary is 30 feet east of the intersection and is walking east at 5ft/sec. How fast is the straight-line distance between them changing at that time? Is it increasing or decreasing?

3. Define the function $f$ by

$$
f(x) = \begin{cases} 
    x^2 \sin \left( \frac{1}{x} \right) & \text{for } x \neq 0 \\
    0 & \text{for } x = 0,
\end{cases}
$$

3.1 Compute $f'(x)$ for $x \neq 0$.
3.2 Show directly from the definition of derivative that $f'(0)$ exists and equals zero.
3.3 Show that $f'(x)$ is discontinuous at $x = 0$.
3.4 Let $g(x) = f(x) + \frac{1}{x}$. Then $g'(x) = f'(x) + \frac{1}{x}$, so by 3.2 $g'(0) = \frac{1}{2}$. Show, nonetheless, that there is no open interval containing 0 on which $g'$ is positive.