Homework to be done by October 7th

Section 2.6: Problems 2, 3, 7, 11, 13, 15.
Section 3.1: Problems 9, 11, 19, 24, 27, 30, 34, 36.
Section 3.2: Problems 27, 30, 54, 57.
Section 3.3: Problems 43, 46, 49, 61, 62.

Another problem to think about: Suppose that \( f \) is differentiable at \( x = a \). Which of the following formulas for \( f'(a) \) are correct?

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\]

\[
f'(a) = \lim_{h \to 0} \frac{f(a) - f(a - h)}{h}
\]

\[
f'(a) = \lim_{h \to 0} \frac{f(a + 2h) - f(a)}{h}
\]

If the formula is incorrect, what is the limit on the right?

Problems to be handed in on October 7th

1. Suppose that \( f \) is a continuous function on \([0,1]\) that takes values in \([0,1]\). (That is \( 0 \leq f(x) \leq 1 \) for \( 0 \leq x \leq 1 \).) Show that there is at least one point \( c \) in \([0,1]\) such that \( f(c) = c \). (\( c \) is called a fixed point of \( f \).) Hint: Consider \( g(x) = f(x) - x \).

2. Let

\[
f(x) = \begin{cases} 
  x^3 - 2 & \text{if } x \leq 2 \\
  Ax + B & \text{if } x > 2,
\end{cases}
\]

where \( A \) and \( B \) are constants.

2.1 What relationship must \( A \) and \( B \) satisfy for \( f \) to be continuous at \( x = 2 \)?

2.2 Precisely one choice of \( A \) and \( B \) will make \( f \) differentiable at \( x = 2 \). Find it.

3. Let \( n \) be a positive integer.

3.1 Prove that if \( 0 \leq a < b \) then \( a^n < b^n \). Hint: Use mathematical induction.

3.2 Prove that every nonnegative real number \( x \) has a unique nonnegative \( n^{th} \) root \( x^{1/n} \). Hint: The existence of \( x^{1/n} \) can be seen by applying the intermediate value theorem to the function \( f(t) = t^n \) for \( t \geq 0 \). The uniqueness follows from 2.1.