Reading: Folland: Chapter 6, sections 3, 4 and 5.

Problems from Folland:

Chapter 6, Section 2: problems 17, 20, 21, 22

Problem 1 Let $1 < p < \infty$. Suppose that $f_n$ converges weakly to $f$ in $L^p$ which we denote by $f_n \rightharpoonup f$ in $L^p$. Show that $\sup_n \|f_n\|_p < \infty$.

Problem 2 Let $f_n \rightharpoonup f$ in $L^2$. Show that $f_n \to f$ in $L^2$ iff $\|f_n\|_2 \to \|f\|_2$.

Problem 3 Let $\Omega \subset \mathbb{R}^n$ be an open bounded connected set. A function $u \in C^2(\Omega)$ is harmonic in $\Omega$ if $\Delta u = 0$. Prove the following statements:

1. **Mean value formula for harmonic functions.** Let $u$ be a harmonic function in $\Omega$. For each ball $B(x, r) \subset \Omega$

   $$u(x) = \int_{\partial B(x, r)} u \, d\sigma = \int_{B(x, r)} u \, dy$$

2. **Strong maximum principle.** Let $u \in C^2(\Omega) \cap C(\bar{\Omega})$ be a harmonic function in $\Omega$, then

   $$\max_{\Omega} u = \max_{\partial \Omega} u.$$