Problem 1. Let $(X, \mathcal{M}, \mu)$ be a measure space. Let $f \in L^r \cap L^\infty$ for some $r \in [1, \infty)$ show that
\[ \lim_{p \to \infty} \|f\|_p = \|f\|_\infty. \]

Problem 2.(Prelim) Suppose that $\sup_n \|f_n\|_p < \infty$ and $f_n \to f$ a.e.
2.1 If $1 < p < \infty$ then $f_n \rightharpoonup f$ in $L^p$, i.e. $\forall g \in L^q$
\[ \int f_n g \to \int fg. \]
(Hint: See problem 20 Chapter 6, section 2 in Folland.)
2.2 Show that the result above is false in general for $p = 1$.

(*) Problem 3. Boundedness of weakly convergent sequences Let $1 < p < \infty$. Assume that $f_n \rightharpoonup f$ in $L^p$. Then
3.1 $\{f_n\}_{n=1}^\infty$ is bounded in $L^p$.
3.2 $\|f\|_p \leq \liminf_{n \to \infty} \|f_n\|_p$.

Problems from Folland
Chapter 6, Section 2 : problems 17, 22