Problem 1. (Prelim) Let \( \{f_n\}_{n \geq 1} \) be a sequence of non-negative functions defined on a measure space \((X, \mathcal{M} \mu)\), such that for every \( n \geq 1 \),
\[
\int_X f_n \, d\mu \leq 1.
\]
Prove that
\[
\limsup_{n \to \infty} (f_n(x))^{1/n} \leq 1 \quad \mu - \text{a.e. } x \in X.
\]

Problem 2. (Prelim) Show, with justification of each step, that
\[
\int_0^1 \left( \sum_{n=1}^{\infty} x^k \frac{\cos(2^k \pi x)}{k} \right) \, dx = \sum_{n=1}^{\infty} \left( \int_0^1 x^k \frac{\cos(2^k \pi x)}{k} \, dx \right).
\]

Problems from Folland:
Chapter 2, Section 3: problems 18, 19, 20, 21, 27, 28 (a-c).