

MATH 301: Practice Midterm 1

1. Consider the polynomial $f(x) = x^2 + 5x + 1$
 - (a) Assuming $f(n)$ is even for some n , prove that $f(n + 1)$ is also even.
 - (b) Is $f(n)$ ever actually even? What's wrong with the inductive "proof" in part (a)?
2. How many divisors does the number 400,000,000 have? Of these, how many end in a zero?
3. If possible, find $x, y \in \mathbb{Z}$ so that $15x + 28y = 1$.
4. Determine whether the following statements are true or false. If true, give a short proof. If false, give an counterexample.
 - (a) If $a|n$ and $b|n$, then $ab|n$.
 - (b) If $\gcd(x, y) = 1$, then $\gcd(x^2, y) = 1$.
5. If $\sigma(n)$ is odd, what can be said about n ?
6. (★) Prove that the number

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

is never an integer for $n \geq 2$.