MATH 301: Midterm 1

1. Find integers $x, y \in \mathbb{Z}$ so that $12x + 29y = 1$.

2. Only one of the following two statements is true:

   Statement A: "The sum of any 4 consecutive integers is divisible by 4."

   Statement B: "The sum of any 5 consecutive integers is divisible by 5."

   For the true statement, provide a proof. For the false statement, give a counterexample.

3. Find the smallest number with exactly 20 divisors.

4. In this problem a, b, and n are positive integers.

   (a) If $a|n$, $b|n$, and $\gcd(a, b) = 1$, prove that $ab|n$.

   (b) Give an example to show that the statement in part (a) is not necessarily true if you remove the "$\gcd(a, b) = 1$" condition.

5. Choose **one** of the following statements to prove (both are true!)

   (a) For any $k > 1$, there are infinitely many n such that $\tau(n) = k$.

   (b) $\tau(n) \leq 2\sqrt{n}$ for all $n \geq 1$.

6. (★) A number n is called abundant if $\sigma(n) > 2n$. Let's call a number n **hyperabundant** if $\sigma(n) > 100n$.

   Do hyperabundant numbers exist? Either prove or disprove the existence of such numbers.