MATH 301: Problem Set 5

1. Make addition and multiplication tables for $\mathbb{Z}/5\mathbb{Z}$ (the integers modulo 5) and $\mathbb{Z}/6\mathbb{Z}$ (the integers modulo 6). Comment on any interesting similarities or differences you observe.

2. Let p be a prime number.

   (a) Prove that if $ab \equiv 0 \pmod{p}$, then $a \equiv 0$ or $b \equiv 0 \pmod{p}$.

   (b) Give an example to show that part (a) is not necessarily true if the modulus is not prime.

3. For how many values of n with $1 \leq n \leq 100$ is $n^2 + 3n + 2$ divisible by 6?

4. Here is a useful divisibility rule for 11: "A number is divisible by 11 if and only if the alternating sum of the digits is divisible by 11." Here alternating sum means you alternatively add and subtract the digits.

   For example, 83952 is divisible by 11 because $8 - 3 + 9 - 5 + 2 = 11$, which is divisible by 11.

   Explain why this divisibility rule works. [Hint: $10 \equiv -1 \pmod{11}$]

5. Prove that the equation $x^3 + y^3 + z^3 = 1796$ has no integer solutions. [Hint: work modulo 9]

6. (★) Prove that any natural number divides some Fibonacci number.