MATH 301: Problem Set 2

1. (a) Use the Euclidean algorithm to find $\gcd(221, 187)$.
   (b) Find integers x and y so that $\gcd(221, 187) = 221x + 187y$.

2. In this problem F_n indicates the nth Fibonacci number.
   (a) Prove the identity $F_n = 5F_{n-4} + 3F_{n-5}$.
   (b) Prove the following interesting property of Fibonacci numbers: if $5|n$, then $5|F_n$. (Hint: part (a) and induction)

3. Prove the following well-known divisibility rule for 4: "If the last two digits of a number form a number that's divisible by 4, then the number is divisible by 4". For example, 65732 is divisible by 4 because 32 is.

   (Hint: you can express your number in the form $n = 100a + b$, where b is the number formed by the last 2 digits)

4. Is it true that $\gcd(x, y) = \gcd(x + y, x - y)$? If so prove it. If not, find a counterexample.

5. The paper you are holding measures 8.5 inches by 11 inches. Using this info, figure out a way to measure out a perfect 1-inch square. No rulers allowed!

6. (★) What are the possible values of $\gcd(n^2 + 1, (n + 1)^2 + 1)$? (You may wish to start by generating some data for small values of n and making a **conjecture**. Then see if you can prove your conjecture.)