

Integer Programming and the Perfect Graph Theorem

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Abstract

The Perfect Graph Theorem is often thought of in combinatorial terms. We define a graph to be perfect if for every induced subgraph, the size of the smallest anti-clique cover (also known as the chromatic number) equals the clique number (the size of the largest clique). The Perfect Graph Theorem then states that a graph is perfect if and only if its complement is perfect. We can then connect these strictly combinatorial definitions to integer programming. For example, we can write integer programs whose maxima and minima extend the idea of chromatic and clique numbers. Likewise, pluperfection extends the definition of perfection using equalities from the corresponding integer programs. We then introduce the Pluperfect Graph Theorem, which is simply an extension of the Perfect Graph Theorem using the above. We show how the Pluperfect Graph Theorem is in fact a special case of a theorem on anti-blocking polyhedra in integer programming by Fulkerson. With this, and various combinatorial results, we complete the proof of the Perfect Graph Theorem.