Lecturer: Rekha Thomas — Due date: Monday, March 4, 2019, in class

Problem Set 8

# 409 - Discrete Optimization

Spring 2019

**Exercise 1**

1. Write down the max matching problem and the min vertex cover problems in a graph $G = ([n], E)$ as integer programs using the vertex-edge incidence matrix $A_G$ of $G$ to express the constraints.
2. Argue that you do not need the upper bound of 1 on the variables in the max matching LP.
3. Write down the dual of an LP of the form $\max\{c^\top x : Ax \leq b, x \geq 0\}$.
4. Write down the dual of the LP relaxation of the max matching problem.
5. Is this dual solving the min vertex cover problem?

**Exercise 2**

A stable set in a graph $G = (V, E)$ is a subset $S \subseteq V$ such that for any two vertices $i, j \in S$, the pair $\{i, j\}$ is not an edge in $E$.

1. Formulate the problem of finding a stable set in $G$ of maximum cardinality as an integer program. (You already did this in a previous homework.)
2. Let $P$ denote the feasible region of the LP relaxation of the integer program in part 1. Prove that if $G$ is bipartite, then $P$ is an integral polytope (i.e., $P = P^I$).

**Exercise 3**

Let $A \in \{0, 1, -1\}^{m \times n}$ be a TU matrix, $b \in \mathbb{Z}^m$ and $c \in \mathbb{Z}^n$. Prove that the primal and dual LPs shown below both have integer optimal solutions.

$$\max\{c^\top x : Ax \leq b\} = \min\{y^\top b : y \geq 0,\ y^\top A = c^\top\}.$$

**Exercise 4**

Give an example of an integer matrix $A$ and integer vector $b$ such that the polyhedron

$$P = \{x : Ax \leq b\}$$

is integral but $A$ is not TU.