

Math 409, Winter 2019 — Problem Set 3

due Monday Jan 28, in class

Exercise 1

Model the minimum cost spanning tree problem in an undirected graph $G = ([n], E)$ as an integer program. Show all steps and write the final model clearly at the end. You will need to use properties of spanning trees to write the constraints.

Exercise 2

Consider the following polyhedron:

$$P = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, -11x + 6y \leq 33, 8x + 5y \leq 40\}.$$

1. Draw the polyhedron P and its integer hull P^I .
2. Write down the inequalities that describe P^I as a polyhedron.
3. What are the optimal solutions of the linear program and integer program over P with respect to the cost function $2x - y$.
4. What are the optimal solutions of the linear program and integer program over P with respect to the cost function y .
5. Calculate the dual optimal solution for the LP with cost function y .
6. For which cost vectors do the IP and LP agree in this example? In each case, write down the optimal solution of the IP/LP.