Lecturer: Rekha Thomas Due date: Monday, May 21, 2017, in class

Problem Set 7

409 - Discrete Optimization

Spring 2018

Exercise 1
A stable set in a graph $G = (V, E)$ is a subset $S \subseteq V$ such that for any two vertices $i, j \in S$, the pair $\{i, j\}$ is not an edge in E.

1. Formulate the problem of finding a stable set in G of maximum cardinality as an integer program.

2. Let P denote the convex hull of incidence vectors of all stable sets in G. Prove that if G is bipartite, then P is an integral polytope (i.e., $P = P^I$).

Exercise 2
A *clique* in a graph G is a subgraph that is a complete graph.

1. Formulate the problem of finding the largest clique in a graph as an integer program.

2. Do you see a connection between the max stable set problem and the max clique problem in a general graph?

Exercise 3
Let $A \in \{0, 1, -1\}^{m \times n}$ be a TU matrix, $b \in \mathbb{Z}^m$ and $c \in \mathbb{Z}^n$. Prove that the primal and dual LPs shown below both have integer optimal solutions.

$$\max\{c^\top x : Ax \leq b\} = \min\{y^\top b : y \geq 0,\ y^\top A = c^\top\}.$$

Exercise 4
Give an example of an integer matrix A and integer vector b such that the polyhedron

$$P = \{x : Ax \leq b\}$$

is integral but A is not TU.