Lecturer: Rekha Thomas Due date: Monday, May 7, 2018, in class

Problem Set 5

409 - Discrete Optimization

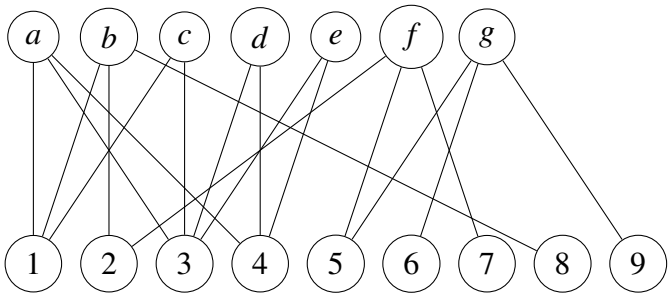
Spring 2018

Exercise 1
Suppose we have k machines $m_1, m_2, \ldots, m_k$ and we need to use them to carry out n jobs $j_1, \ldots, j_n$. Each machine can do certain jobs. For example, here is a table that shows which machine can be used for which job for $k = n = 5$.

	j_1	j_2	j_3	j_4	j_5
m_1	X	X			X
m_2	X	X	X	X	
m_3	X	X			
m_4		X			
m_5		X			

On a single day a machine can only be used to do one job. Find an assignment of jobs to machines so that you maximize the number of jobs that get done.

Exercise 2
Find a minimum vertex cover in the following graph.



Exercise 3
Consider the linear program

$$\begin{aligned} \max\ & x_1 - 2x_2 && (P)\\ & x_1 + x_2 \leq 4\\ & x_1 - 3x_2 \leq 6\\ & -2x_1 - x_2 \leq -3\\ & 4x_1 - 3x_2 \leq 15 \end{aligned}$$

a) Draw the feasible region of LP (P).

b) Determine the optimum solution x^* by inspecting the picture.

c) State the dual program (to (P)).

d) Find the optimum solution to the dual. Check that it is feasible and the objective function value matches the objective function value for the dual.
Hint: Your knowledge about duality will tell you very quickly how the dual solution has to look like. You do not need to run the simplex algorithm to find the dual solution!

Exercise 4
Let $x_1, \ldots, x_k \in \mathbb{R}^n$ be vectors and let $K := \text{conv}\{x_1, \ldots, x_k\}$. Prove that if the origin $\mathbf{0} = (0, \ldots, 0)$ is not in K, then there is a vector $c \in \mathbb{R}^n$ with $cx_i \geq 1$ for all $i = 1, \ldots, k$.
Hint: Apply the hyperplane separation theorem.