MATH 409 LECTURE 5
BASICS OF GRAPH THEORY

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Definition 1.

- An **undirected** graph $G = (V, E)$ is the pair of sets V and E where V is the set of **vertices** of G and E the set of (undirected) **edges** of G. An edge $e \in E$ is a set $\{i, j\}$ where $i, j \in V$. We write $G = (V(G), E(G))$ if there are multiple graphs being considered.
- The graph G is **simple** if it has no parallel edges (i.e., more that one edge $\{i, j\}$ for $i, j \in V$), and no loops (edges of the form $\{i, i\}$ for $i \in V$).
- If the edges of G are precisely the $\binom{|V|}{2}$ pairs $\{i, j\}$ for every pair $i, j \in V$ then G is the **complete graph** on V. The complete graph on $n = |V|$ vertices is denoted as K_n.
- A **subgraph** of $G = (V(G), E(G))$ is a graph $H = (V(H), E(H))$ where $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ with the restriction that if $\{i, j\} \in E(H)$ then $i, j \in V(H)$.
- If $V' \subseteq V(G)$, then the subgraph **induced** by V' is the graph $(V', E(V'))$ where $E(V')$ is the set of all edges in G for which both vertices are in V'.
- A subgraph H of G is a **spanning subgraph** of G if $V(H) = V(G)$.
- Given $G = (V, E)$ we can define subgraphs obtained by **deletions** of vertices or edges.
  - If $E' \subseteq E$ then $G \backslash E' := (V, E \backslash E')$.
  - If $V' \subseteq V$ then $G \backslash V' = (V \backslash V', E \backslash \{\{i, j\} \in E : i \in V' \text{ or } j \in V'\})$.
- A **path** in $G = (V, E)$ in a sequence of vertices and edges $v_0, e_1, v_1, e_2, v_2, e_3, \ldots, e_k, v_k$, such that for $i = 0, \ldots, k$, $v_i \in V$, $e_i \in E$ where $e_i = (v_{i-1}, v_i)$. This is called a (v_0, v_k)-**path**. The **length** of the path is the number of edges in the path which equals k. This path is **closed** if $v_0 = v_k$. A closed path is a **circuit** if the vertices are distinct (except the first and the last). A **cycle** in G is a collection of circuits in G.

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- A graph G is **connected** if there is a path in the graph between any two vertices of the graph.
- A graph G is **acyclic** is it contains no circuits as subgraphs.
- An acyclic graph is called a **forest**. A connected forest is a **tree**.
- A spanning subgraph H of a connected graph G is a **spanning tree** if it is a tree (i.e., a connected graph with no cycles).