# Math 408 Midterm Exam Winter 2008

### Name:

1. There are **FOUR** questions in all. Answer all questions.

2. There is a blank sheet at the end that you can tear out and use for scratch work. This sheet does not need to be submitted with the test. If you need extra sheets please ask.

### 3. READ THE QUESTIONS CAREFULLY.

4. Show all your work to get full credit.

#### 5. No notes or calculators are allowed during the test.

Problem #1	Problem $#2$	Problem #3	Problem #4	Total points

**Problem 1** (5 points) Prove the triangle inequality for vectors in  $\mathbf{R}^n$  which says that

 $||\mathbf{x} + \mathbf{y}|| \le ||\mathbf{x}|| + ||\mathbf{y}||, \ \forall \ \mathbf{x}, \mathbf{y} \in \mathbf{R}^n.$ 

**Problem 2** (5 points) Graph the region defined as

$$D = \{ (x_1, x_2) \in \mathbf{R}^2 : |x_1| \le 1, \ e^{(x_1 + x_2)} \ge 1, \ \cos(x_1) \ge x_2 \}.$$

Label axes, important intersection points and all constraints clearly and mark the set D clearly.

Problem 3 (8 points) Consider the NLP:

min 
$$x_1^2 + (x_2 - 2)^2 + 100 \sin(x_1 + x_2)$$
  
s.t.  $(x_1, x_2) \in D$ 

where D is as defined in Problem (2). Using either Theorem 3.2.1 or Corollary 3.2.1 decide whether this NLP will have a global optimum. Explain clearly. **Problem 4** (12 points) Are the following statements true or false? Give short clear reasons or a counterexample to support your answer.

(i) The function  $f := x_1^2 + x_2^4 - 2x_3 + 2x_1x_3 - 4x_1x_2$  is not coercive.

(ii) The following problem could have a local optimum that is not global.

min  $300x_1 + 15x_2 - 75x_3 - x_{10}$ s.t.  $15x_1 + 75x_2 - 300x_3 + 8x_7 \le 500$  $x_1 \ge 0, \dots, x_{10} \ge 0$ 

(iii) Every set of the form  $D = {\mathbf{x} \in \mathbf{R}^n : g(\mathbf{x}) \le 0}$  is closed where g is a function in n variables.

## SCRATCH PAPER