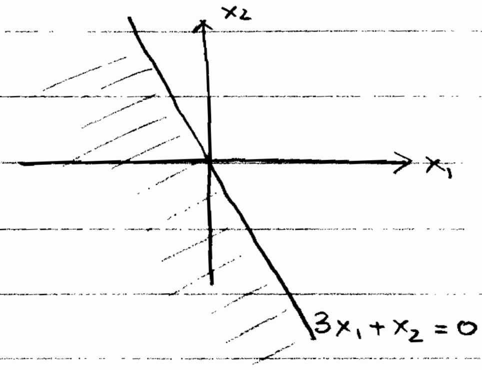
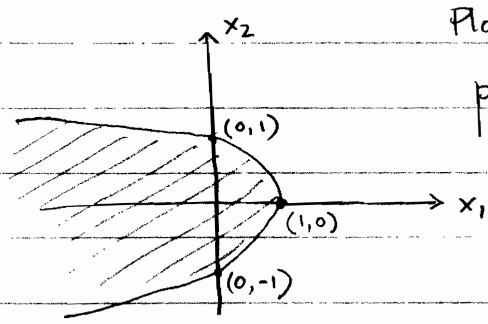
1

Math 408 Homework 2 Solutions

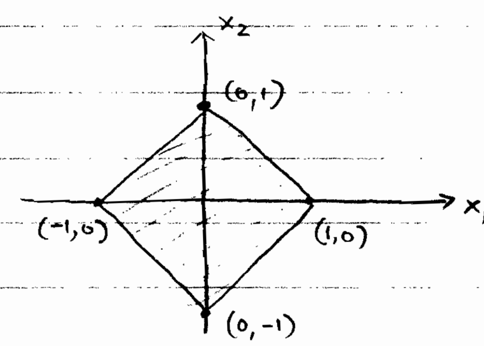


2.1.1 $D_1 = \{(x_1, x_2) \in \mathbb{R}^2 : e^{3x_1 + x_2} \le 1\}$

$D_1 = \{(x_1, x_2) : x_1 + x_2^2 \le 1\}$



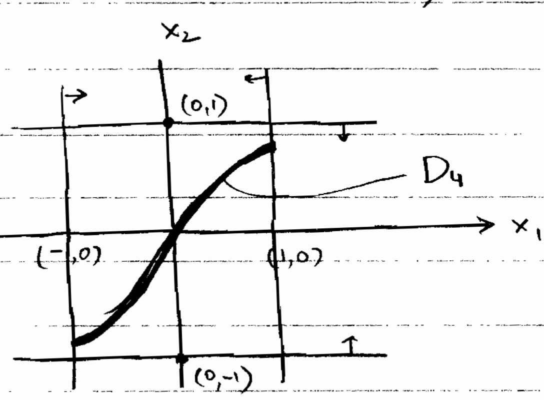
Plot $x_1 + x_2^2 = 1$ which is the parabola you see. Pick the part where the $\le$ is satisfied



$D_3 = \{(x_1, x_2) : |x_1| + |x_2| \le 1\}$

$D_4 = \{(x_1, x_2) : |x_1| \le 1, \ |x_2| \le 1$

$\sin(x_1) = x_2\}$



$\sin(1) < \sin \pi/2 = 1$

$\sin(-1) > \sin -\pi/2 = -1$

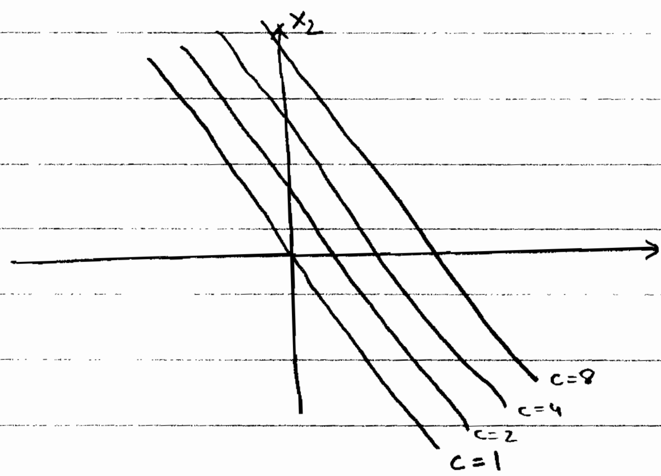
2.1.2 a) $e^{3x_1 + x_2} = c$ $c = 1, 2, 4, 8$

$e^{3x_1 + x_2} = c$

$\Rightarrow \log_e(e^{3x_1+x_2}) = \log_e c$

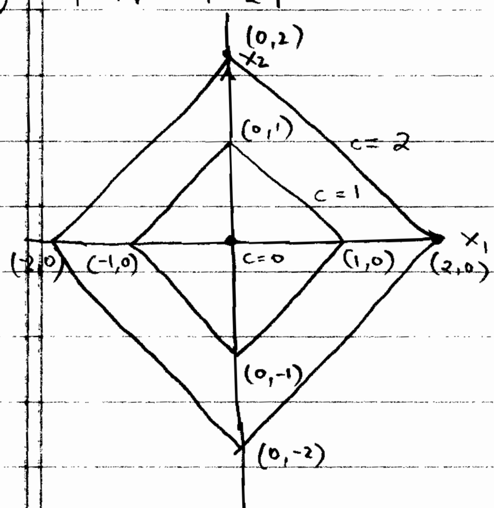
$3x_1 + x_2 = \log_e c$

$3x_1 + x_2 = \begin{cases} 0 & (c=1) \\ <1 & (c=2) \\ >1 & (c=4) \\ >'' & (c=8) \end{cases}$



b) $x_1 + x_2^2 = c$ $c = -1, 0, 1$ (saw in class)

c) $|x_1| + |x_2| = c$ $c = 0, 1, 2$



2.1.3 $x \in \mathbb{R}^n$ $A \in \mathbb{R}^{n\times n}$ $n \geq 2$

$(Ax)^T = x^T A^T$ by property of transpose in the notes

x^2 makes no sense since x is not a number

$x^T x$ is a number

xx^T is a $n \times n$ matrix

Ax^T - no sense - x/A

$\frac{1}{x}$ makes no sense

$x^T A x$ ok - number

$\sin(\|x\|)$ ok since $\|x\| \in \mathbb{R}$

xAx not ok.

xA does not make sense since $x \in \mathbb{R}^{n\times 1}$ & $A \in \mathbb{R}^{n \times n}$

Ax ok - $n \times 1$ matrix / vector

b) $A \in \mathbb{R}^{m \times p}$ $B \in \mathbb{R}^{m \times n}$ $C \in \mathbb{R}^{n \times p}$ $D \in \mathbb{R}^{\ell \times p}$

$D(A + \alpha BC)^T$:

$\alpha BC \in \mathbb{R}^{m \times p} \Rightarrow (A + \alpha BC) \in \mathbb{R}^{m \times p}$

$\Rightarrow (A + \alpha BC)^T \in \mathbb{R}^{p \times m} \Rightarrow D(A + \alpha BC)^T \in \mathbb{R}^{\ell \times m}$

$D(A + \alpha BC)^T = D(A^T + \alpha (BC)^T) = DA^T + \alpha DC^T B^T$

2.1.4 (1) $\|x\|^2 = \sum_{i=1}^n x_i^2 \geq 0 \Rightarrow \|x\| = \sqrt{\sum_{i=1}^n x_i^2} \geq 0$

(2) $\|x\| = 0 \iff \|x\|^2 = 0 \iff \sum_{i=1}^n x_i^2 = 0$

$\iff x_i = 0 \ \forall i = 1, \dots, n \iff x = 0$.

(3) $\|\alpha x\| = \sqrt{\sum_{i=1}^n (\alpha x_i)^2} = \sqrt{\alpha^2 \sum_{i=1}^n x_i^2} = |\alpha| \sqrt{\sum_{i=1}^n x_i^2} = |\alpha| \|x\|$

(4) $\|x+y\|^2 = (x+y)^T(x+y) = x^T x + 2x^T y + y^T y = \|x\|^2 + \|y\|^2 + 2x^T y$

$\leq \|x\|^2 + \|y\|^2 + 2|x^T y| \leq \|x\|^2 + \|y\|^2 + 2\|x\| \|y\|$

$= (\|x\| + \|y\|)^2 \quad \therefore \|x+y\| \leq \|x\| + \|y\|$.

(5) Let $p = \max_{i=1,\dots,n} |x_i|$

RHS $\|x\| = \sqrt{\sum_{i=1}^n x_i^2} \leq \sqrt{\sum_{i=1}^n p^2} = \sqrt{np^2} = \sqrt{n}\, p$

LHS $\|x\| = \sqrt{\sum_{i=1}^n x_i^2} \geq \sqrt{p^2} = p$

2.1.5. Easy.