Math 408: Homework 1 Solutions

1 (i) $x_1 = \dfrac{b_1 - x_4 - x_5}{2}$ $\quad x_2 = \dfrac{b_2 - x_4 - x_6}{2}$ $\quad x_3 = \dfrac{b_3 - x_5 - x_6}{2}$

(i) There is no (b_1, b_2, b_3) for which system is infeasible since the functions on the right hand side are linear and can be evaluated for every value of (b_1, b_2, b_3)

(ii) Infinitely many since x_4, x_5, x_6 can take arbitrary values

(iii) Set $x_4 = x_5 = x_6 = 0$ say then $x_1 = 5$ $x_2 = 5$, $x_3 = 5$
$\therefore (5,5,5,0,0,0)$ is a sol$^{\underline{n}}$

(iv) Plane of dimension 3 in $\mathbb{R}^6$ since each equation cuts dimension by one and the 3 equations are linearly independent

(v) 3 (see above)

(vi) 0, 1, ∞

2. The only constraints that need to be modified are the $x_i = 0, 1$ constraints. Model with $x_i(x_i - 1) = 0$
or $x_i^2 - x_i = 0$

$\therefore$ $\max\ cx$

$Ax \le b$

$x_i^2 - x_i = 0 \quad \forall\, i = 1, .., n$

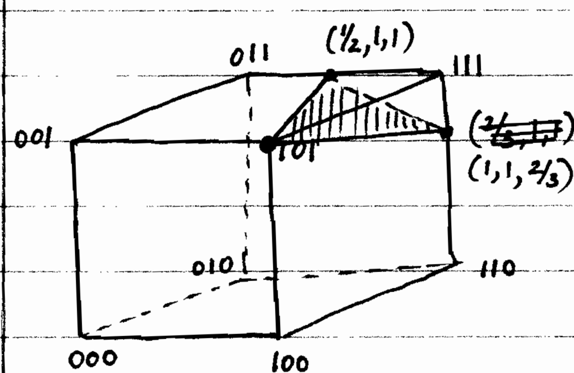
Note: If x_i could take values $p_1, p_2, \cdots, p_k$ we model with $(x_i - p_1)(x_i - p_2) \cdots (x_i - p_k) = 0$
This eq$^{\underline{n}}$ has at most k roots and by construction $p_1, .., p_k$ are k of the roots, so it has exactly $p_1, .., p_k$ as roots.

3. Assume $V = \{1,..,n\}$.
Assign variables x_i $i = 1,..,n$ (one for each node)
that takes values $x_i = 1$ (if node is in stable set)
$x_i = 0$ (otherwise

$\therefore x_i = 0, 1$ $i = 1,..,n$
To get a stable set, we need to prevent both vertices in an edge from being chosen
i.e $x_i + x_j \leq 1$ or $x_i x_j = 0$.

$\therefore$ Max stable set problem: $\max \sum_{i=1}^{n} x_i$
s.t $x_i + x_j \leq 1$ $\forall\, ij \in E$ ~~$i=1,..,n$~~
$x_i^2 - x_i = 0$ $i = 1,..,n$

or $\max \sum_{i=1}^{n} x_i$
s.t $x_i x_j = 0$ $\forall\, ij \in E$
$x_i^2 - x_i = 0$ $\forall\, i = 1,..,n$

4.


max
$x_1 + x_2 + 3x_3$

$2x_1 + x_2 + 3x_3 \leq 5$ violates $(1,1,1)$.
The equation is $2x_1 + x_2 + 3x_3 = 5$
It cuts the edge $x_1 = 1$ $x_3 = 1$ at $x_2 = 0$

It cuts the edge $x_2 = 1$, $x_3 = 1$ at $x_1 = \frac{1}{2}$

It cuts the edge $x_1 = 1$, $x_2 = 1$ at $x_3 = 2/3$

$\therefore$ Extreme pts ~~are~~ with objective function values are:

(0,0,0)	0
(1,0,0)	1
(0,1,0)	1
(1,1,0)	2
(0,0,1)	3
(1,0,1)	4
(0,1,1)	4
(½,1,1)	4½
~~(2/3,1,1)~~	
(1,1,2/3)	5

Optimal solⁿ $(1, 1, 2/3)$ //