Math 408 Nonlinear Optimization (Winter 2008)

- Course web page at http://www.math.washington.edu/~thomas
- Review of linear algebra and multivariable calculus on web page (linked from J. Burke's page) (main assignment for this week)
- Homework 1 will be posted on Wed 1/9. Due in class on Wed 1/16.

What is a nonlinear program?

General NLP

$$\begin{aligned} &\text{maximize/minimize } f(x_1, \dots, x_n) \\ &\text{s.t.} \quad g_i(x_1, \dots, x_n) \geq 0 \quad i = 1, \dots, r \qquad \text{(inequalities)} \\ &\qquad\ h_j(x_1, \dots, x_n) = 0 \quad j = 1, \dots, s \qquad \text{(equations)} \end{aligned}$$

where f, g_i, h_j are non-linear/linear functions in the variables $x_1, \dots, x_n$ from $\mathbb{R}^n \longrightarrow \mathbb{R}$.

(Fields other than $\mathbb{R}$ are possible but the focus in this class will be on $\mathbb{R}$ to emphasize applications of NLPs)

NLP - nonlinear optimization problem

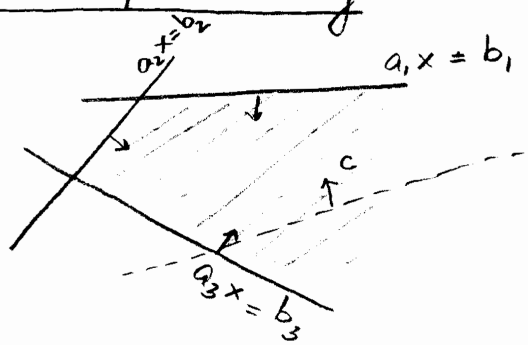
Note: NLP includes linear programs (LP)

Ex 1 Linear programs

$$\begin{array}{ll} \max & c_1 x_1 + \cdots + c_n x_n \\ \text{s.t.} & a_{11} x_1 + \cdots + a_{1n} x_n \le b_1 \\ & a_{21} x_1 + \cdots + a_{2n} x_n \le b_2 \\ & \vdots \\ & a_{m1} x_1 + \cdots + a_{mn} x_n \le b_m \end{array} \iff \begin{array}{ll} \max & c \cdot x \\ \text{s.t.} & Ax \le b \end{array}$$

where
$c = (c_1, \dots, c_n) \in \mathbb{R}^n$
$A = (a_{ij}) \in \mathbb{R}^{m \times n}$
$b = (b_1, \dots, b_m) \in \mathbb{R}^m$

Structure / Geometry



feasible region is a "polyhedron"
convex set
Optimal sol$^{\underline{n}}$ found by "sliding" the $cx = ?$ plane over the feasible region.

3 possibilities for an LP:

- LP
  - feasible
    - bounded ($\iff \exists$ optimal sol$^{\underline{n}}$)
    - unbounded (opt val $= \infty$, no finite opt sol$^{\underline{n}}$)
  - infeasible: $\{x : Ax \le b\} = \emptyset$

Algorithms for LP.

- Simplex Method (Math 407) (not known to be a polynomial time alg)
- Nonlinear methods (interior point methods etc) polynomial time

LP Duality

LP: $\max\ cx$ st $Ax \le b$ dual LP: $\min\ yb$, $yA = c$, $y \ge 0$

LP feasible and bounded $\iff$
dual LP feasible and bounded. In this case

$$\max_{Ax \le b} cx = \min_{\substack{yA = c \\ y \ge 0}} yb$$

LP used widely in practise. Approximates many complicated problems and can be solved in millions of variables

Software packages CPLEX