

MATH 407 Homework on LP Geometry

Consider the following linear program:

$$\begin{array}{rllll} \text{maximize} & & x_1 & + & x_2 \\ \text{subject to} & 3x_1 & + & 4x_2 & \leq & 37 \\ & -x_1 & + & x_2 & \leq & 4 \\ & x_1 & & & \leq & 7 \\ & x_1 & \geq & & & 0 \\ & x_2 & \geq & & & 0 \end{array}$$

(1) Provide a graphical solution to the above LP.

(2) Name the hyperplanes corresponding to the nonnegativities as  $H_1$  and  $H_2$  and the rest as  $H_3, H_4, H_5$  to match the indices of the slack variables in those constraints. Indicate which is which.

(3) Run the simplex method on this LP using dictionaries. At each dictionary, record the following information:

1. The basic feasible solution carried by this dictionary.
2. The solution in  $x_1, x_2$  coordinates corresponding to this basic feasible solution.
3. The hyperplanes that are active at this solution. “Active” means that the solution is on that hyperplane.
4. Which variables are positive in this basic feasible solution? Explain them in terms of the hyperplanes  $H_i$ .
5. Choose a variable to enter the basis and perform the minimum ratio test to force a variable to leave the basis. Explain what happened in terms of the geometry.

(4) Write down the dictionary that corresponds to the vertices  $(0, 4)$  and  $(7, 0)$ .

(5) Write all five coordinates of the feasible solution  $x_1 = 1, x_2 = 1$  of this problem. Is this a basic feasible solution? Why or why not?