Math 308 N Test Prep Homework

DUE WEDNESDAY JAN 30

(1) For each of the situations described below, **give an example** (if it's possible) or **explain why it's not possible**.

(a) A set of vectors that does not span $\mathbb{R}^3$. After adding one more vector, the set does span $\mathbb{R}^3$.

(b) A set of vectors that are linearly dependent. After adding one more vector, the set becomes linearly independent.

(c) A set of vectors in $\mathbb{R}^3$ with the following properties (four possibilities):

spans $\mathbb{R}^3$, linearly independent	spans $\mathbb{R}^3$, linearly dependent
doesn't span $\mathbb{R}^3$, linearly independent	doesn't span $\mathbb{R}^3$, linearly dependent

For each case that is *possible*, how many vectors could be in the set? (State any constraints, as in "there must be at least..." or "at most...")

(e) A system of equations with a unique solution. After adding another equation to the system, the new system has infinitely-many solutions.

(f) * A system of equations without any solutions. After deleting an equation, the system has infinitely-many solutions.

(2) (after 2.3) (Geometry Question) Consider the infinite system of linear equations in two variables given by $ax + by = 0$ where (a, b) moves along the unit circle in the plane.

Recall that the vector (a, b) is the normal to the line with equation $ax + by = 0$. In other words, (a, b) is perpendicular to the line.

(a) How many solutions does this system have?

(b) How many linearly independent equations in the above system give you the same set of solutions? Write down two separate such linear systems, in vector form. (A system of equations in this situation is linearly independent if their normal vectors are linearly independent.)

(c) What happens to the infinite linear system if you add to it the equation $0x + 0y = 0$?

(d) What happens to the infinite linear system if one of the equations slightly perturbs to $ax + by = c$ where c is a small positive number?

Explain all your answers in words.

(3) Determine whether $\text{span}\left\{\begin{pmatrix}-1\\4\\2\end{pmatrix}, \begin{pmatrix}2\\3\\1\end{pmatrix}\right\} = \text{span}\left\{\begin{pmatrix}1\\7\\3\end{pmatrix}, \begin{pmatrix}-4\\5\\3\end{pmatrix}\right\}$. Explain your logic and computations.

(4) (a) Find a vector in $\mathbb{R}^3$ not in the span of $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$.

(b) Does the vector you found along with the two vectors from part (a) span $\mathbb{R}^3$? Explain your answer.

(c) Let $A = \begin{pmatrix} -2 & 1 \\ 1 & -3 \\ 3 & 1 \end{pmatrix}$. For which $b \in \mathbb{R}^3$ does the system $Ax = b$ have a solution? When there is a solution, it is necessarily unique?