Practice Quiz – Math 308 E

Recall that if we have m vectors $u_1, u_2, \ldots, u_m$ in $\mathbb{R}^n$, then we can form the matrix A whose columns are $u_1, \ldots, u_m$. Let B be the echelon form of A. All the questions below are based on such a matrix B. Most questions have a yes/no answer, but I am mostly interested in your reasons for the answer. Give full reasons for all answers.

Suppose we are given the following matrix B:

$$\begin{pmatrix} 3 & 0 & -1 & 5 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(1) What is n?
(2) What is m?
(3) Are $u_1, \ldots, u_m$ linearly independent?
(4) Does $\{u_1, \ldots, u_m\}$ span $\mathbb{R}^n$?
(5) Looking at B can you write down a subset of the original set $\{u_1, \ldots, u_m\}$ that would be guaranteed to be linearly independent?
(6) Is there a subset of the original set $\{u_1, \ldots, u_m\}$ that would be guaranteed to span $\mathbb{R}^n$?
(7) Write down a $b \in \mathbb{R}^n$ for which $Bx = b$ does not have a solution.
(8) Write down a $b \in \mathbb{R}^n$ for which $Bx = b$ has a solution.
(9) Write down a $b \in \mathbb{R}^n$ for which $Bx = b$ has a unique solution.
(10) Is there a new vector $w \in \mathbb{R}^n$ that you could add to the set $\{u_1, \ldots, u_m\}$ to guarantee that $\{u_1, \ldots, u_m, w\}$ will span $\mathbb{R}^n$?
(11) Is there a column of B that is in the span of the rest? If so, find it.
(12) Looking at B do you see a u_i that is in the span of the others? How can you identify it?
(13) Assuming that no row of A was a zero row, how many planes are being used to cut out the solutions of $Ax = 0$?
(14) Looking at B, how many planes are needed at the minimum to cut out the solution set of $Ax = 0$?
(15) If your answers to the last two questions are different, why is there a difference?
(16) Put B into reduced echelon form.
(17) Write down a non-zero solution of $Ax = 0$ if you can.
(18) How many free variables are there in the set of solutions to $Ax = b$ when there is a solution?
(19) If you erased the last row of zeros in B then would the columns of the resulting matrix be linearly independent?
(20) Can you add rows to B to make the columns of the new matrix linearly independent? If yes, give an example of the new matrix you would construct.