# Math 308 E Test Prep Problems - Week #3

(1) (after 2.3) (Geometry Question) Consider the infinite system of linear equations in two variables given by $ax + by = 0$ where $(a, b)$ moves along the unit circle in the plane.

(a) How many solutions does this system have?
(b) How many linearly independent equations in the above system give you the same set of solutions? Write down two separate such linear systems, in vector form. (A system of equations in this situation is linearly independent if their normal vectors are linearly independent.)
(c) What happens to the infinite linear system if you add to it the equation $0x + 0y = 0$?
(d) What happens to the infinite linear system if one of the equations slightly perturbs to $ax + by = c$ where $c$ is a small positive number?
Explain all your answers in words.

**Solution by Groups A11, B11, C11 – due in class on Friday 1/19**

(2) Determine whether $\text{span}\left\{\begin{pmatrix}-1\\4\\2\end{pmatrix}, \begin{pmatrix}2\\3\\1\end{pmatrix}\right\} = \text{span}\left\{\begin{pmatrix}1\\7\\3\end{pmatrix}, \begin{pmatrix}-4\\5\\3\end{pmatrix}\right\}$. Explain your logic and computations.

**Solution by Groups A12, B12, C12 – due in class on Monday 1/22**

(3) (a) Find a vector in $\mathbb{R}^3$ not in the span of $\begin{pmatrix}-2\\1\\3\end{pmatrix}, \begin{pmatrix}1\\-3\\1\end{pmatrix}$.

(b) Does the vector you found along with the two vectors from part (a) span $\mathbb{R}^3$? Explain your answer.

(c) Let $A = \begin{pmatrix}-2 & 1\\1 & -3\\3 & 1\end{pmatrix}$. For which $b \in \mathbb{R}^3$ does the system $Ax = b$ have a solution? When there is a solution, it is necessarily unique?

**Solution by Groups A13, B13, C13 – due in class on Monday 1/22**